CSE 312 Section 2

Intro to Discrete Probability

Administrivia

Announcements & Reminders

- Office Hours
  - Offered every day! Mostly in-person, a few zoom options
  - Times posted on the calendar on the course website

- HW1
  - Was due yesterday, Wednesday 4/3 @ 11:59pm
  - Late deadline Saturday 4/6 @ 11:59pm (max of 3 late days per homework)

- HW2
  - Released on the course website
  - Due Wednesday 4/10 @ 11:59pm

Homework

- Submissions
  - LaTeX (highly encouraged)
    - overleaf.com
    - template and LaTeX guide posted on course website!
  - Word Editor that supports mathematical equations
  - Handwritten neatly and scanned
- Homework will typically be due on Wednesdays at 11:59pm on Gradescope
- Each assignment can be submitted a max of 72 hours late
- You have **8 late days total** to use throughout the quarter
  - Anything beyond that will result in a deduction on further late assignments

Review

Any lingering questions from this last week?

Kahoot!

https://kahoot.it/?pin=1389366&refer_method=link

Problem 6 – Powers and Divisibility

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Prove that there exist two powers of 7 whose difference is divisible by 2003. (You may want to use the Pigeonhole principle.)

Work on this problem with the people around you, and then we'll go over it together!

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Prove that there exist two powers of 7 whose difference is divisible by 2003. (You may want to use the Pigeonhole principle.)

HINT: think about what are the pigeons, and what are the pigeonholes?

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6 – Powers and Divisibility

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Problem 10 – Balls from an Urn

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Say an urn contains one red ball, one blue ball, and one green ball. (Other than for their colors, balls are identical.) Imagine we draw two balls with replacement, i.e., after drawing one ball, with put it back into the urn, before we draw the second one. (In particular, each ball is equally likely to be drawn.)

a) Give a probability space describing the experiment.
b) What is the probability that both balls are red? (Describe the event first, before you compute its probability.)
c) What is the probability that at most one ball is red?
d) What is the probability that we get at least one green ball?
e) Repeat b)-d) for the case where the balls are drawn without replacement, i.e., when the first ball is drawn, it is not placed back from the urn.

Work on this problem with the people around you, and then we'll go over it together!

10 – Balls from an Urn

An urn contains one red ball, one blue ball, and one green ball. Draw two balls with replacement.

a) Give a probability space describing the experiment.

10 – Balls from an Urn

An urn contains one red ball, one blue ball, and one green ball. Draw two balls with replacement.

b) What is the probability that both balls are red? (Describe the event first, before you compute its probability.)

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An urn contains one red ball, one blue ball, and one green ball.

e) Repeat b)-d) for the case where **the balls are drawn without replacement.**

a) Does the probability space change? If so, what is it now?

a) What is the probability that both balls are red?

a) What is the probability that at most one ball is red?

a) What is the probability that we get at least one green ball?

Problem 11 – Weighted Die

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Consider a weighted die such that

- $\mathbb{P}(1) = \mathbb{P}(2)$
- $\mathbb{P}(3) = \mathbb{P}(4) = \mathbb{P}(5) = \mathbb{P}(6)$
- $\mathbb{P}(1) = 3 \cdot \mathbb{P}(3)$

What is the probability that the outcome is 3 or 4?

Work on this problem with the people around you, and then we'll go over it together!

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Problem 2 - Subsubset

2 – Subsubset

- Let $[n] = \{1,2, \dots, n\}$ denote the first n natural numbers. How many (ordered) pairs of subsets (A, B) are there such that $A \subseteq B \subseteq [n]$?

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2 – Subsubset

- Let $[n] = \{1,2, \dots, n\}$ denote the first n natural numbers. How many (ordered) pairs of subsets (A, B) are there such that $A \subseteq B \subseteq [n]$?

Problem 4 – GREED INNIT

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Find the number of ways to rearrange the word “INGREDIENT”, such that no two identical letters are adjacent to each other. For example, “INGREEDINT” is invalid because the two E’s are adjacent.

Repeat the question for the letters “AAAAABBB”.

Work on this problem with the people around you, and then we’ll go over it together!

4 – GREED INNIT

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Repeat the question for the letters “AAAAABBB”.

Problem 12 – Shuffling Cards

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We have a deck of cards, with 4 suits, with 13 cards in each. Within each suit, the cards are ordered Ace > King > Queen > Jack > 10 > · · · > 2. Also, suppose we perfectly shuffle the deck (i.e., all possible shuffles are equally likely). What is the probability the first card on the deck is (strictly) larger than the second one?

Work on this problem with the people around you, and then we'll go over it together!

12 – Shuffling Cards

We have a deck of cards, with 4 suits, with 13 cards in each. Within each suit, the cards are ordered Ace > King > Queen > Jack > 10 > · · · > 2. Also, suppose we perfectly shuffle the deck (i.e., all possible shuffles are equally likely). What is the probability the first card on the deck is (strictly) larger than the second one?

Problem 5 - Friendships

5 – Friendships

Show that in any group n people there are two who have an identical number of friends within the group. (Friendship is bi-directional – i.e., if A is friend of B, then B is friend of A – and nobody is a friend of themselves.) Solve in particular the following two cases individually:

a) Everyone has at least one friend.

b) At least one person has no friends.

Work on this problem with the people around you, and then we'll go over it together!

5 – Friendships

a) Everyone has at least one friend.

5 – Friendships

b) At least one person has no friends.

That's All, Folks!

Thanks for coming to section this week!
Any questions?