Union Bound

For any events E, F
$\mathbb{P}(\boldsymbol{E} \cup \boldsymbol{F}) \leq \mathbb{P}(\boldsymbol{E}) + \mathbb{P}(\boldsymbol{F})$

(Multiplicative) Chernoff Bound

Let $X_1, X_2, \ldots, X_n$ be *independent* Bernoulli random variables.
Let $X = \sum X_i$, and $\mu = \mathbb{E}[X]$. For any $0 \leq \delta \leq 1$

$$\mathbb{P}(X \geq (1+\delta)\mu) \leq \exp\left(-\frac{\delta^2 \mu}{3}\right) \text{ and } \mathbb{P}(X \leq (1-\delta)\mu) \leq \exp\left(-\frac{\delta^2 \mu}{2}\right)$$

29

Hoeffding's Inequality

Hoeffding's Inequality

Let $X_1, X_2, \ldots, X_n$ be *independent* RVs, each with range $[0,1]$.
Let $\bar{X} = \sum X_i / n$, and $\mu = \mathbb{E}[\bar{X}]$. For any $t \geq 0$

$$\mathbb{P}(|\bar{X} - \mathbb{E}[\bar{X}]| \geq t) \leq 2\exp(-2nt^2)$$

How close will we be with *n=1000 with probability at least* .95*?*

$|X - \mathbb{E}[X]| \geq t$ if and only if $|Y - \mathbb{E}[Y]| \geq 2t$.

30

Doing Better With Randomness

You don't really need to know who was cheating. Just how many people were.

Here's a protocol:

Please flip a coin.

If the coin is heads, or you have ever cheated, please tell me "heads"

If the coin is tails and you have not ever cheated, please tell me "tails"

12

But will it be accurate?

But we've lost our data haven't we? People answered a different question. Can we still estimate how many people cheated?

Suppose you asked 100 people the "heads/tails" question, and 60 people said "heads." What do you predict would be the number of people who cheated on a partner?

Can you generalize your idea for n people polled, and X the number of people that said "heads"?

15