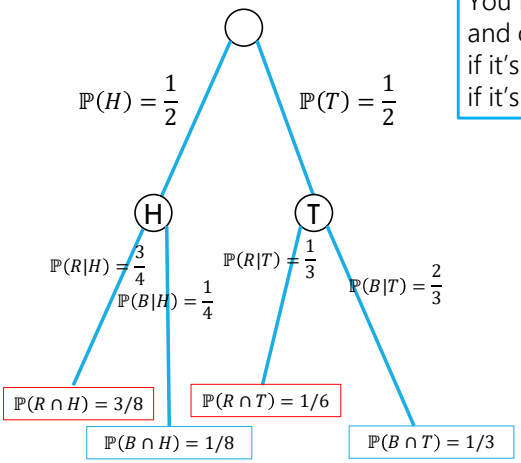
Updated Sequential Processes



You have three red marbles and one blue marble in your left pocket, and one red marble and two blue marbles in your right pocket.
if it's heads, you'll draw a marble (uniformly) from your left pocket,
if it's tails, you'll draw a marble (uniformly) from your right pocket.

For sequential processes with probability, at each step multiply by
$\mathbb{P}(\text{next step} \mid \text{all} \cap \text{prior} \cap \text{steps})$

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Independence for 3 or more events

For three or more events, we need two kinds of independence

Pairwise Independence

Events $A_1, A_2, \ldots, A_n$ are pairwise independent if
$\mathbb{P}(\boldsymbol{A_i} \cap \boldsymbol{A_j}) = \mathbb{P}(\boldsymbol{A_i}) \cdot \mathbb{P}(\boldsymbol{A_j})$ **for all $\boldsymbol{i, j}$**

Mutual Independence

Events $A_1, A_2, \ldots, A_n$ are mutually independent if
$\mathbb{P}(\boldsymbol{A_{i_1}} \cap \boldsymbol{A_{i_2}} \cap \cdots \cap \boldsymbol{A_{i_k}}) = \mathbb{P}(\boldsymbol{A_{i_1}}) \cdot \mathbb{P}(\boldsymbol{A_{i_2}}) \cdots \mathbb{P}(\boldsymbol{A_{i_k}})$
for every subset $\{\boldsymbol{i_1}, \boldsymbol{i_2}, \ldots, \boldsymbol{i_k}\}$ of $\{\boldsymbol{1}, \boldsymbol{2}, \ldots, \boldsymbol{n}\}$.

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Application 1: Medical Tests

Helping Doctors and Patients Make Sense of Health Statistics

A researcher posed the following scenario to a group of 160 doctors:

Assume you conduct a disease screening using a standard test in a certain region. You know the following information about the people in this region:

The probability that a person has the disease is 1% (prevalence)

If a person has the disease, the probability that she tests positive is 90% (sensitivity)

If a person does not have the disease, the probability that she nevertheless tests positive is 9% (false-positive rate)

A person tests positive. She wants to know from you whether that means that she has the disease for sure, or what the chances are. What is the best answer?

A. The probability that she has the disease is about 81%.
B. Out of 10 people with a positive test, about 9 have the disease.
C. Out of 10 people with a positive test, about 1 have the disease.
D. The probability that she has the disease is about 1%

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Implicitly defining Ω

We've often skipped an explicit definition of Ω.

Often $|\Omega|$ is infinite, so we really couldn't write it out (even in principle).

How would that happen?

Flip a fair coin (independently each time) until you see your first tails. what is the probability that you see at least 3 heads?

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