Independence

Independence

Two events A, B are independent if
$$\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$$

You'll sometimes see this called "statistical independence" to emphasize that we're talking about probabilities (not, say, physical interactions).

If A, B both have non-zero probability then

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B) \Leftrightarrow \mathbb{P}(A|B) = \mathbb{P}(A) \Leftrightarrow \mathbb{P}(B|A) = \mathbb{P}(B)$$

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Chain Rule

We defined conditional probability as: $\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$

Which means $\mathbb{P}(A \cap B) = \mathbb{P}(A|B)\mathbb{P}(B)$

Chain Rule

$$\mathbb{P}(A_1 \cap A_2 \cap \cdots \cap A_n)$$
$$= \mathbb{P}(A_n|A_1 \cap \cdots \cap A_{n-1}) \cdot \mathbb{P}(A_{n-1}|A_1 \cap \cdots \cap A_{n-2}) \cdots \mathbb{P}(A_2|A_1) \cdot \mathbb{P}(A_1)$$

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Conditional Independence Example

You have two coins. Coin A is fair, coin B comes up heads with probability 0.85.

You will roll a (fair) die, if the result is odd flip coin A twice (independently); if the result is even flip coin B twice (independently)

Let C_1 be the event "the first flip is heads", C_2 be the event "the second flip is heads", O be the event "the die was odd"

Are C_1 and C_2 independent? Are they independent conditioned on O?

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Flipping the conditioning

What about $\mathbb{P}(T|B)$?

You have three red marbles and one blue marble in your left pocket, and one red marble and two blue marbles in your right pocket.
if it's heads, you'll draw a marble (uniformly) from your left pocket,
if it's tails, you'll draw a marble (uniformly) from your right pocket.

Pause, what's your intuition?

Is this probability

A. less than ½

B. equal to ½

C. greater than ½

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