Conditioning Practice

Red die 6 conditioned on sum 7

Red die 6 conditioned on sum 9

Sum 7 conditioned on red die 6

Take a few minutes to work on this with the people around you! (also on your handout)

	D2=1	D2=2	D2=3	D2=4	D2=5	D2=6
D1=1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1.6)
D1=2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
D1=3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
D1=4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
D1=5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
D1=6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

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Conditional Probability

Conditional Probability

For an event B, with $\mathbb{P}(B) > 0$,
the "Probability of A conditioned on B" is
$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$$

Just like with the formal definition of probability, this is pretty abstract. It does accurately reflect what happens in the real world.

If $\mathbb{P}(B) = 0$, we can't condition on it (it can't happen! There's no point in defining probabilities where we know B has not happened) – $\mathbb{P}(A|B)$ is **undefined** when $\mathbb{P}(B) = 0$.

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Conditioning

Let S be the event that the Scale alerts you

Let G be the event your bar has a Golden ticket.

What conditional probabilities are each of these?

Willy Wonka has placed golden tickets on 0.1% of his Wonka Bars.
If the bar you weigh **does** have a golden ticket, the scale will alert you 99.9% of the time.
If the bar you weigh does not have a golden ticket, the scale will (falsely) alert you only 1% of the time.
You pick up a bar and it alerts, what is the probability you have a golden ticket?
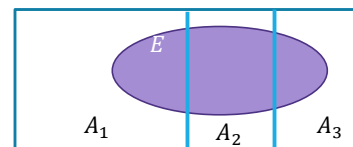
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LTP and Bayes

Law of Total Probability

Let $A_1, A_2, \ldots, A_k$ be a **partition** of Ω.
For any event E,

$$\mathbb{P}(E) = \sum_{\textbf{all } i} \mathbb{P}(E|A_i)\mathbb{P}(A_i)$$

Ω, split into partition A_1, A_2, A_3 with event E inside.

A_1 A_2 A_3 E

Bayes' Rule

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$$

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