One More Counting Rule

You're going to buy one-dozen donuts (i.e., 12 donuts)

There are chocolate, strawberry, coconut, blueberry, and lemon (i.e. five types)

How many different donut boxes can you buy?

Consider two boxes the same if they contain the same number of every kind of donut (order doesn't matter)

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We've seen lots of ways to count

Sum rule (split into disjoint sets)

Product rule (use a sequential process)

Combinations (order doesn't matter)

Permutations (order does matter)

Principle of Inclusion-Exclusion

Complementary Counting

"Stars and Bars" $\binom{n+k-1}{k-1}$

Niche Rules (useful in very specific circumstances)

- Binomial Theorem
- Pigeonhole Principle

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Uniform Probability Space

The most common probability measure is the **uniform** probability measure. In the uniform measure, for every event E

$\mathbb{P}(E) = \frac{|E|}{|\Omega|}$.

Let your sample space be all possible outcomes of a sequence of 100 coin tosses. Assign the uniform measure to this sample space. What is the probability of the event “there are exactly 50 heads?

A. $\binom{100}{50}/2^{100}$

B. $1/101$

C. $1/2$

D. $1/2^{50}$

E. There is not enough information in this problem.

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Probability Space

Probability Space

A (discrete) probability space is a pair $(\Omega, \mathbb{P})$ where:
Ω is the sample space
$\mathbb{P}: \Omega \to [0,1]$ is the probability measure.
$\mathbb{P}$ satisfies:

- $\mathbb{P}(x) \geq 0$ **for all** x
- $\sum_{x \in \Omega} \mathbb{P}(x) = 1$
- **If** $E, F \subseteq \Omega$ **and** $E \cap F = \emptyset$ **then** $\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F)$

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