CSE 312 Section 6

Continuous RVs

Administrivia

Announcements & Reminders

- Quiz 5
  - Tomorrow, 12pm
  - Continuous Random Variable
- HW5
  - Released last night
  - Written is due Wednesday 8/6 @ 11:59pm

Review & Questions

Any lingering questions from this last week?

Each week in section, we'll be reviewing the main concepts from this week and putting them into action by going through some practice problems together. But before we get into that review, we'll try to start off each section with some time for you to ask questions. Was anything particularly confusing this week? Is there anything we can clarify before we dive into the review? This is your chance to clear things up!

Problem 1 – Content Review Questions

1 – Content Review

a) 0

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b) $\int_{-\infty}^{k} f_X(x)dx$

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a) 0

b) $\int_{-\infty}^{k} f_X(x)dx$

c) $\frac{d}{dk} F_X(k)$

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a) 0
b) $\int_{-\infty}^{k} f_X(x)dx$
c) $\frac{d}{dk}F_X(k)$
d) True

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e) True

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a) 0
b) $\int_{-\infty}^{k} f_X(x)dx$
c) $\frac{d}{dk}F_X(k)$
d) True
e) True
f) Time (real number) to the first success with λ as average number of successes per minute

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a) 0
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c) $\frac{d}{dk} F_X(k)$
d) True
e) True
f) Time (real number) to the first success with λ as average number of successes per minute
g) True

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c) $\frac{d}{dk} F_X(k)$
d) True
e) True
f) Time (real number) to the first success with λ as average number of successes per minute
g) True
h) False

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a) 0
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c) $\frac{d}{dk}F_X(k)$
d) True
e) True
f) Time (real number) to the first success with λ as average number of successes per minute
g) True
h) False
i) True

Problem 2 – Uniform2

2 – Uniform2

Robbie decided he wanted to create a “new” type of distribution that will be famous, but he needs some help. He knows he wants it to be continuous and have uniform density, but he needs help working out some of the details. We’ll denote a random variable X having the “Uniform-2” distribution as $X \sim Uniform2(a, b, c, d)$, where $a < b < c < d$. We want the density to be non-zero in $[a, b]$ and $[c, d]$, and zero everywhere else. Anywhere the density is non-zero, it must be equal to the same constant.

(a) Find the probability density function, $f_X(x)$. Be sure to specify the values it takes on for every point in $(-\infty, \ \infty)$. (Hint: use a piecewise definition).
(b) Find the cumulative distribution function, $F_X(x)$. Be sure to specify the values it takes on for every point in $(-\infty, \ \infty)$. (Hint: use a piecewise definition).

Work on this problem with the people around you, and then we’ll go over it together!

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(a) Find the probability density function, $f_X(x)$. Be sure to specify the values it takes on for every point in $(-\infty, \infty)$. (Hint: use a piecewise definition).

$$f_X(x) = \begin{cases} \frac{1}{(b-a)+(d-c)}, & x \in [a, b] \cup [c, d] \\ 0, & otherwise \end{cases}$$

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$$F_X(x) = \begin{cases} 0, & x \in (-\infty, a) \\ \dfrac{(x - a)}{(b - a) + (d - c)}, & x \in [a, b) \\ \dfrac{(b - a)}{(b - a) + (d - c)}, & x \in [b, c) \\ \dfrac{(b - a) + (x - c)}{(b - a) + (d - c)}, & x \in [c, d) \\ 1, & x \in [d, \infty) \end{cases}$$

Problem 6 – Throwing a dart

6 – Throwing a dart

Consider the closed unit circle of radius r, i.e. $S=\{ (x, y) : x^2 + y^2 \leq r^2 \}$. Suppose we throw a dart onto this circle and are guaranteed to hit it, but the dart is equally likely to land anywhere in S. Concretely, this means that the probability that the dart lands in any particular area of size A is equal to $\frac{A}{Area\ of\ the\ whole\ circle}$. The density outside the unit circle is 0.

Let X be the distance the dart lands from the center. What is the CDF and PDF of X? What is $E[X]$ and $Var(X)$?

Work on this problem with the people around you, and then we'll go over it together!

6 – Throwing a dart

- Since $F_X(x)$ is the probability that the dart lands inside the circle of radius x, that probability is the area of a circle of radius x divided by the area of the circle of radius r (i.e., $\frac{\pi \cdot x^2}{\pi \cdot r^2}$). Thus, our CDF looks like

$$F_X(x) = \begin{cases} 0 & x < 0 \\ x^2/r^2 & 0 \le x \le r \\ 1 & x > r \end{cases}$$

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To find the PDF we just need to take the derivative of the CDF, which give us the following:

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Using the definition of expectation we get:

$$E[X] = \int_{-\infty}^{\infty} x f_X(x) dx = \int_0^r \frac{2x^2}{r^2} dx = \frac{2}{3x^2}\left(x^3 \Big|_0^r\right) = \frac{2}{3}r$$

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We know that $Var(X) = E[X^2] - E[X]^2$:

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x) dx = \int_0^r \frac{2x^3}{r^2} dx = \frac{2}{4r^2}\left(x^4 \Big|_0^r\right) = \frac{1}{2}r^2$$

Plugging this into our variance equation gives:

$$\Rightarrow Var(X) = \frac{1}{2}r^2 - \left(\frac{2}{3}r\right)^2 = \frac{1}{18}r^2$$

Problem 10 – Normal Questions

10 – Normal Questions

a) Let X be a normal random with parameters $\mu = 10$ and $\sigma^2 = 36$. Compute $\mathbb{P}(4 < X < 16)$.

b) Let X be a normal random variable with $\mu = 5$. If $\mathbb{P}(X > 9) = 0.2$, approximately what is $Var(X)$?

c) Let X be a normal random variable with mean 12 and variance 4. Find the value of c such that $\mathbb{P}(X > c) = 0.1$.

Work on this problem with the people around you, and then we'll go over it together!

10 – Normal Questions

a) Let X be a normal random with parameters $\mu = 10$ and $\sigma^2 = 36$. Compute $\mathbb{P}(4 < X < 16)$.

Let $Z = \frac{X-10}{6}$. Then $Z \sim N(0,1)$. So,

$$\mathbb{P}(4 < X < 16) = \mathbb{P}\left(\frac{4-10}{6} < Z < \frac{16-10}{6}\right) = \mathbb{P}(-1 < Z < 1) = \Phi(1) - \Phi(-1) = 0.68268$$

10 – Normal Questions

b) Let X be a normal random variable with $\mu = 5$. If $\mathbb{P}(X > 9) = 0.2$, approximately what is $Var(X)$?

Let $\sigma^2 = Var(X)$. Then,

$$\mathbb{P}(X > 9) = \mathbb{P}\left(\frac{X-5}{\sigma} > \frac{9-5}{\sigma}\right) = 1 - \Phi\left(\frac{4}{\sigma}\right) = 0.2$$

$$\Phi\left(\frac{4}{\sigma}\right) = 0.8$$

Looking up the z-table, we get $\frac{4}{\sigma} = 0.845$, so $\sigma \approx 4.73$. Therefore, $Var(X) \approx 22.4$.

10 – Normal Questions

c) Let X be a normal random variable with mean 12 and variance 4. Find the value of c such that $\mathbb{P}(X > c) = 0.1$.

$$\mathbb{P}(X > c) = \mathbb{P}\left(\frac{X-12}{2} > \frac{c-12}{2}\right) = 1 - \Phi\left(\frac{c-12}{2}\right) = 0.1$$

$$\Phi\left(\frac{c-12}{2}\right) = 0.9$$

Looking up the z-table, we get $\frac{c-12}{2} = 1.29$, so $c \approx 14.58$.

That’s All, Folks!

Thanks for coming to section this week!
Any questions?