CSE 312 Section 3

Conditional Probability

Administrivia

Announcements & Reminders

- HW1
  - Grades released on gradescope – check your submission to read comments
  - Regrade requests opened and close after a week

- HW2
  - Was due yesterday, Wednesday 7/9 @ 11:59pm
  - Late deadline Friday 7/11 @ 11:59pm **(with meaningful attempt submitted yesterday)**

- HW3
  - Released on the course website
  - Due Wednesday 7/16 @ 11:59pm

Announcements & Reminders

- Quiz 2
  - Tomorrow at 12pm!
  - 15 mins long
  - Topics covered last week and this week
  - Probability, Conditional Probability, Bayes' theorem, Independence, etc.

Review

Any lingering questions from this last week?

Each week in section, we'll be reviewing the main concepts from this week and putting them into action by going through some practice problems together. But before we get into that review, we'll try to start off each section with some time for you to ask questions. Was anything particularly confusing this week? Is there anything we can clarify before we dive into the review? This is your chance to clear things up!

What have we learned?

- Conditional Probability
- Bayes' Theorem
- Independence

Problem 1 – Content Review

Problem 4 – Game Show

4 – Game Show

Corrupted by their power, the judges running the popular game show America's Next Top Mathematician have been taking bribes from many of the contestants. During each of two episodes, a given contestant is either allowed to stay on the show or is kicked off. If the contestant has been bribing the judges, she will be allowed to stay with probability 1. If the contestant has not been bribing the judges, she will be allowed to stay with probability 1/3, independent of what happens in earlier episodes. Suppose that 1/4 of the contestants have been bribing the judges. The same contestants bribe the judges in both rounds.

4 – Game Show

2 rounds, contestants bribe in both rounds. If contestant has been bribing , allowed to stay with probability 1. If contestant has not been bribing, allowed to stay with probability 1/3, independent of earlier. Suppose that 1/4 of the contestants have been bribing the judges.

a) If you pick a random contestant, what is the probability that she is allowed to stay during the first episode?
b) If you pick a random contestant, what is the probability that she is allowed to stay during both episodes?
c) If you pick a random contestant who was allowed to stay during the first episode, what is the probability that she gets kicked off during the second episode?
d) If you pick a random contestant who was allowed to stay during the first episode, what is the probability that she was bribing the judges?

Work on this problem with the people around you, and then we'll go over it together!

Problem 5 – Parallel Systems

5 – Parallel Systems

A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

a) What is the probability the system is functioning?

b) If the system is functioning, what is the probability that component 1 is working?

c) If the system is functioning and component 2 is working, what is the probability that component 1 is working?

Work on this problem with the people around you, and then we'll go over it together!

Problem 13 – Balls from an Urn – Take 2

13 – Balls from an Urn – Take 2

Say an urn contains three red balls and four blue balls. Imagine we draw three balls without replacement. (You can assume every ball is uniformly selected among those remaining in the urn.)

a) What is the probability that all three balls are all of the same color?
b) What is the probability that we get more than one red ball given the first ball is red?

Work on this problem with the people around you, and then we'll go over it together!

Problem 9 – Dependent Dice Duo

9 – Dependent Dice Duo

This problem demonstrates that independence can be "broken" by conditioning.

Let D_1 and D_2 be the outcomes of two independent rolls of a fair die. Let E be the event "$D_1 = 1$", F be the event "$D_2 = 6$", and G be the event "$D_1 + D_2 = 7$". Even though E and F are independent, show that

$$\mathbb{P}(E \cap F | G) \neq \mathbb{P}(E|G)\mathbb{P}(F|G)$$

Work on this problem with the people around you, and then we'll go over it together!

Problem 6 – Allergy Season

6 – Allergy Season

In a certain population, everyone is equally susceptible to colds. The number of colds suffered by each person during each winter season ranges from 0 to 4, with probability 0.2 for each value (see table below). A new cold prevention drug is introduced that, for people for whom the drug is effective, changes the probabilities as shown in the table. Unfortunately, the effects of the drug last only the duration of one winter season, and the drug is only effective in 20% of people, independently.

Number of colds	No drug / ineffective	Drug effective
0	0.2	0.4
1	0.2	0.3
2	0.2	0.2
3	0.2	0.1
4	0.2	0.0

6 – Allergy Season

The drug is only effective in 20% of people, independently.

Number of colds	No drug / ineffective	Drug effective
0	0.2	0.4
1	0.2	0.3
2	0.2	0.2
3	0.2	0.1
4	0.2	0.0

a) Sneezy decides to take the drug. Given that he gets 1 cold that winter, what is the probability that the drug is effective for Sneezy?

b) The next year he takes the drug again. Given that he gets 2 colds in this winter, what is the updated probability that the drug is effective for Sneezy?

c) Why is the answer to (b) the same as the answer to (a)?

Work on this problem with the people around you, and then we'll go over it together!

Problem 3 – Marbles in Pockets

3 – Marbles in Pockets

Aleks has 5 blue and 3 white marbles in her left pocket, and 4 blue and 4 white marbles in her right pocket. If he transfers a randomly chosen marble from left pocket to right pocket without looking, and then draws a randomly chosen marble from her right pocket, what is the probability that it is blue?

Work on this problem with the people around you, and then we'll go over it together!

Problem 7 – A Game

7 – A Game

Howard and Jerome are playing the following game: A 6-sided die is thrown and each time it's thrown, regardless of the history, it is equally likely to show any of the six numbers.

- If it shows 5, Howard wins.
- If it shows 1, 2, or 6, Jerome wins.
- Otherwise, they play a second round and so on.

What is the probability that Jerome wins on the 4th round?

Work on this problem with the people around you, and then we'll go over it together!

That’s All, Folks!

Thanks for coming to section this week!
Any questions?