CSE 312 Section 2

Intro to Discrete Probability

Administrivia

Announcements & Reminders

- Office Hours
  - Offered every day!
  - Times posted on the calendar on the course website

- HW1
  - Was due yesterday, Wednesday 7/2
  - Late deadline Saturday 7/4 (with meaningful attempt submitted yesterday)

- HW2
  - Released on the course website
  - Due Wednesday 7/9

Review

What have we learned?

- Principle of Inclusion-Exclusion
- Stars and Bars
- Pigeonhole principle
- Discrete Probability
- Conditional Probability

Any lingering questions from this last week?

Problem 1 – Content Review

Problem 2 – A Team and a captain

6 – Powers and Divisibility

Give a combinatorial proof of the following identity:

$$n\binom{n-1}{r-1} = \binom{n}{r} r$$

Hint: Consider two ways to choose a team of size r out of a set of size n and a captain of the team (who is also one of the team members)

Work on this problem with the people around you, and then we'll go over it together!

Problem 6 – Powers and Divisibility

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Prove that there exist two powers of 7 whose difference is divisible by 2003. (You may want to use the Pigeonhole principle.)

Work on this problem with the people around you, and then we'll go over it together!

6 – Powers and Divisibility

Prove that there exist two powers of 7 whose difference is divisible by 2003. (You may want to use the Pigeonhole principle.)

HINT: think about what are the pigeons, and what are the pigeonholes?

Work on this problem with the people around you, and then we'll go over it together!

Problem 10 – Balls from an Urn

10 – Balls from an Urn

Say an urn contains one red ball, one blue ball, and one green ball. (Other than for their colors, balls are identical.) Imagine we draw two balls with replacement, i.e., after drawing one ball, with put it back into the urn, before we draw the second one. (In particular, each ball is equally likely to be drawn.)

a) Give a probability space describing the experiment.
b) What is the probability that both balls are red? (Describe the event first, before you compute its probability.)
c) What is the probability that at most one ball is red?
d) What is the probability that we get at least one green ball?
e) Repeat b)-d) for the case where the balls are drawn without replacement, i.e., when the first ball is drawn, it is not placed back from the urn.

Work on this problem with the people around you, and then we'll go over it together!

10 – Balls from an Urn

An urn contains one red ball, one blue ball, and one green ball. Draw two balls with replacement.

a) Give a probability space describing the experiment.

10 – Balls from an Urn

An urn contains one red ball, one blue ball, and one green ball. Draw two balls with replacement.

b) What is the probability that both balls are red? (Describe the event first, before you compute its probability.)

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An urn contains one red ball, one blue ball, and one green ball.

e) Repeat b)-d) for the case where **the balls are drawn without replacement.**

   a) Does the probability space change? If so, what is it now?

   a) What is the probability that both balls are red?

   a) What is the probability that at most one ball is red?

   a) What is the probability that we get at least one green ball?

Problem 11 – Weighted Die

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- Consider a weighted die such that
- $\mathbb{P}(1) = \mathbb{P}(2)$
- $\mathbb{P}(3) = \mathbb{P}(4) = \mathbb{P}(5) = \mathbb{P}(6)$
- $\mathbb{P}(1) = 3 \cdot \mathbb{P}(3)$

What is the probability that the outcome is 3 or 4?

Work on this problem with the people around you, and then we'll go over it together!

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Problem 2 - Subsubset

2 – Subsubset

- Let $[n] = \{1,2,\ldots,n\}$ denote the first n natural numbers. How many (ordered) pairs of subsets (A, B) are there such that $A \subseteq B \subseteq [n]$?

Work on this problem with the people around you, and then we'll go over it together!

Problem 4 – GREED INNIT

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Find the number of ways to rearrange the word “INGREDIENT”, such that no two identical letters are adjacent to each other. For example, “INGREEDINT” is invalid because the two E’s are adjacent.

Repeat the question for the letters “AAAAABBB”.

Work on this problem with the people around you, and then we’ll go over it together!

4 – GREED INNIT

Repeat the question for the letters “AAAAABBB”.

Problem 12 – Shuffling Cards

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We have a deck of cards, with 4 suits, with 13 cards in each. Within each suit, the cards are ordered Ace > King > Queen > Jack > 10 > · · · > 2. Also, suppose we perfectly shuffle the deck (i.e., all possible shuffles are equally likely). What is the probability the first card on the deck is (strictly) larger than the second one?

Work on this problem with the people around you, and then we'll go over it together!

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Problem 5 - Friendships

5 – Friendships

- Show that in any group n people there are two who have an identical number of friends within the group. (Friendship is bi-directional – i.e., if A is friend of B, then B is friend of A – and nobody is a friend of themselves.) Solve in particular the following two cases individually:

a) Everyone has at least one friend.

b) At least one person has no friends.

Work on this problem with the people around you, and then we'll go over it together!

5 – Friendships

a) Everyone has at least one friend.

5 – Friendships

b) At least one person has no friends.

That’s All, Folks!

Thanks for coming to section this week!
Any questions?