Section Midterm: Midterm Review

1. True or False?

Let A and B be events in the same sample space that each have non-zero probability. For each of the following statements, state whether it is always true, always false, or it depends on information not given.

(a) If A and B are mutually exclusive, then they are independent.

(b) If A and B are independent, then they are mutually exclusive.

(c) If $\mathbb{P}(A) = \mathbb{P}(B) = 0.75$, then A and B are mutually exclusive.

(d) If $\mathbb{P}(A) = \mathbb{P}(B) = 0.75$, then A and B are independent.

2. Question 2

Given any set of 18 integers, show that one may always choose two integers so that their difference is divisible by 17.

3. Count the Solutions 2

Consider the following inequality: $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 \leq 70$. A solution to this inequality over the non-negative integers is a choice of a non-negative integer for each of the 6 variables $a_1, a_2, a_3, a_4, a_5, a_6$ that satisfies the inequality. To be different, two solutions have to differ on the value assigned to some a_i. How many different solutions are there to the inequality?

4. Question 4

You roll 3 fair dice, each with a different numbers of faces: die 1 has six faces (numbered $1 \dots 6$), die 2 has eight faces (numbered $1 \dots 8$), and die 3 has twelve faces (numbered $1 \dots 12$). Let the random variable X be the sum of the three values rolled. What is $\mathbb{E}[X]$?

5. Question 5

How many integers in $\{1, 2, \dots, 360\}$ are divisible by one or more of the numbers 2, 3 and 5?

6. Question 6

Suppose a special deck has 4 suits with 5 cards in each suit. It is shuffled well and then dealt into 5 piles of 4 cards each. Let E_i refer to the event that pile i has exactly one spade. Compute the probability $\mathbb{P}(E_1 \cap E_2 \cap E_3 \cap E_4 \cap E_5)$

7. Question 7

You are trying to diagnose the probability that a patient with a positive blood sugar test result has diabetes, even though she is in a low risk group. The probability for women in this group having diabetes is 0.8%. 90% of women with diabetes will test positive in the blood sugar test. 7% of women without diabetes will test positive in the blood sugar test. Your patient tests positive in the blood sugar test. What is the probability that she has diabetes?

8. Question 8

A very long multiple choice exam has 4 choices for each questions. Charlie has studied enough so that he know the correct anser for $\frac{1}{2}$ of the questions; for an additional $\frac{1}{4}$ of the questions he can eliminate one choice and chooses randomly and uniformly among the other three, and for the remaining $\frac{1}{4}$ of the questions he chooses randomly and uniformly among all four answers.

As the teacher, you want to determine how many answers the student actually knows. For a randomly chosen question, if Charlie answers it correctly, what is the probability he knew the answer?

9. Question 9

The space shuttle has 6 O-rings: these were invloved in the Challenger disaster. When the space hsuttle is launched, each O-ring has a probability of failure of 0.0137, independent of whether other O-rings fail.

(a) What is the probability that, during 23 launches, no O-ring will fail, but that at least one O-ring will fail during the 24th launch?

(b) What is the probability that no O-ring fails during 24 launches?

10. Question 10

Suppose you record the birthdays of a large group of people, one at a time, until you have found a person whose birthday matches your own birthday. What is the probability that it takes exactly 20 people for this to occur? Assume that there are 365 possible birthdays and each one is equally probable for a randomly chosen person.

11. Question 11

Two fair 6-sided dice are thrown n times in succession.

(a) Compute the probability that double 6 (i.e., 6 on each die) appears at least once in the n throws.

(b) How large does n need to be to make this probability at least $\frac{1}{2}$?

12. Question 12

You are working on a difficult passage from a new piece you are learning on the piano. You wish to play it correctly 4 times before stopping for the day. If your probability of playing it correctly on each attempt is $\frac{2}{3}$, and the attempts are independent (unfortunately!), what is the probability that you have to play it at least 8 times?

13. Cash???

The probability that a customer pays with cash is 40%, independent of other customers. Find the probability that the 12th customer to arrive at the cashier is the 8th one that pays with cash.

14. Question 14

Let X be the outcome of rolling a fair 6-sided die once. Let Y be the sum of the outcomes of rolling the same die n times independently.

(a) Compute $\mathbb{E}[X]$.

(b) Compute $Var(X)$ and the standard deviation σ of X.

(c) Compute $\mathbb{E}[Y]$.

(d) Compute $Var(Y)$

15. Lucky Bomb Defusal

You are in a tense bomb-defusal scenario with three wires: Red (R), Yellow (Y), and Blue (B). There is a single 'lucky' wire that will defuse the bomb, the other two will cause it to explode.

Captured, the bomber has promised to help, albeit deceitfully. If you pretend to cut a wire, the bomber will reveal a non-lucky (incorrect) wire from the remaining two. If both remaining wires are non-lucky they will pick randomly.

Suppose that you pretend to cut wire R. The bomber then reveals that the lucky wire is not wire B. Given this information, should you cut wire R or wire Y?

Let $\overline{B}$ be the event that the bomber Reveals B to be the incorrect choice.

Let R be the event that the 'lucky'/correct wire is R.

Let Y be the even that the 'lucky'/correct wire is Y.

(a) Find P($\overline{B}$)

(b) Find the probability that the lucky wire is red given B.

(c) Find the probability that the lucky wire is yellow given B.

(d) Which wire should you cut?

Expert profiler Manav Rao, has informed you of one of the bombers 'tells'. When you pretend to cut the 'lucky' wire, the bomber scratches their chin $\frac{3}{5}$ of the time. When you pretend to cut a different wire, the bomber scratches their chin $\frac{1}{10}$ of the time.

Let $\overline{B}$ be the event that the bomber reveals B to be the incorrect choice and scratches their chin.

Let R be the event that the 'lucky'/correct wire is R.

Let Y be the even that the 'lucky'/correct wire is Y.

(a) Find the probability that the lucky wire is red given B.

(b) Find the probability that the lucky wire is yellow given B.

(c) Which wire should you cut?