Example

Suppose you flip a coin independently 10 times, and you see

HTTTHHTHHH

What is your estimate of the probability the coin comes up heads?

1

Coin flips is easier

$\mathcal{L}(\text{HTTTHHTHHH}; \theta) = \theta^6(1-\theta)^4$

$\ln(\mathcal{L}(\text{HTTTHHTHHH}; \theta) = ?$

2

Summary

Given: an event E (usually n i.i.d. samples from a distribution with unknown parameter θ).

1. Find likelihood $\mathcal{L}(E;\theta)$

Usually $\prod \mathbb{P}(x_i;\theta)$ for discrete and $\prod f(x_i;\theta)$ for continuous

2. Maximize the likelihood. Usually:

A. Take the log (if it will make the math easier)

B. Set the derivative to 0 and solve

C. Use the second derivative test to confirm you have a maximizer

3

Variance

$$\ln\left(\mathcal{L}\left(x_i;\theta_\mu,\theta_{\sigma^2}\right)\right) = \sum_{i=1}^{n} \ln\left(\frac{1}{\sqrt{\theta_{\sigma^2} 2\pi}}\right) - \frac{1}{2}\cdot\frac{\left(x_i-\theta_\mu\right)^2}{\theta_{\sigma^2}}$$

Take the partial derivative with respect to θ_{σ^2}. It'll be easier if you apply some log and exponent rules first.

$\log(x^y) = y\cdot\log(x)$.

$\log(ab) = \log(a)+\log(b)$.

$\frac{1}{\sqrt{a}} = a^{-1/2}$

4