LTE

Let $\boldsymbol{X}, \boldsymbol{Y}$ be discrete random variables, then

$$\mathbb{E}[\boldsymbol{X}] = \sum_{\boldsymbol{y} \in \Omega_{\boldsymbol{Y}}} \mathbb{E}[\boldsymbol{X} | \boldsymbol{Y} = \boldsymbol{y}] \mathbb{P}(\boldsymbol{Y} = \boldsymbol{y})$$

You will flip 2 (independent, fair coins). Call the number of heads X. Then (independently of the coin flips) draw an exponential random variable Y from the distribution $\mathrm{Exp}(X + 1)$.

What is $\mathbb{E}[Y]$?

1

Different dice

Find these values

$p_{V|U}(2|1) =$

$p_{U|V}(1|2) =$

$p_{U|V}(4|1) =$

$p_{U,V}$	U=1	U=2	U=3	U=4
V=1	1/16	0	0	0
V=2	2/16	1/16	0	0
V=3	2/16	2/16	1/16	0
V=4	2/16	2/16	2/16	1/16

2

Covariance

We sometimes want to measure how "intertwined" X and Y are – how much knowing about one of them will affect the other.

If X turns out "big" how likely is it that Y will be "big" how much do they "vary together"

Covariance

$$\mathbf{Cov}(\mathbf{X},\mathbf{Y}) = \mathbb{E}[(\boldsymbol{X} - \mathbb{E}[\boldsymbol{X}])(\boldsymbol{Y} - \mathbb{E}[\boldsymbol{Y}])] = \mathbb{E}[\boldsymbol{XY}] - \mathbb{E}[\boldsymbol{X}]\mathbb{E}[\boldsymbol{Y}]$$

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Covariance, Another example

Let X be a Bernoulli RV with probability p of success.

Let $Y = X$ (Y is X, not an iid copy, literally the same experiment)

Let $Z = -X$

Let W be an independent Bernoulli, indentically distributed to X

Find

$\text{Cov}(X, Y)$, $\text{Cov}(X, Z), \text{Cov}(X, W)$

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