

Geometric Distribution

$X \sim \text{Geo}(p)$

p is the probability of success for one trial.

X is the number of trials needed to see the first success.

$$p_X(k) = (1-p)^{k-1}p \text{ for } k \in \{1,2,3, \dots\}$$

$$F_X(k) = 1 - (1-p)^k \text{ for } k \in \mathbb{N}$$

$$\mathbb{E}[X] = \frac{1}{p}$$

$$\text{Var}(X) = \frac{1-p}{p^2}$$

Some other uses:

How many bits can we write before one is incorrect?

How many questions do you have to answer until you get one right?

How many times can you run an experiment until it fails for the first time?

1

Formally...

Let X be the total number of flips needed, Y be the flips after the second.

$$\mathbb{P}(Y = k | X \geq 3) = ?$$

...

Which is $p_X(k)$.

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Try it

More generally, run independent trials with probability p . How many trials do you need for r successes?

What's the pmf?

What's the expectation and variance (hint: linearity)

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Scenario: Hypergeometric

You have an urn with N balls, of which K are purple. You are going to draw balls out of the urn **without** replacement.

If you draw out n balls, what is the probability you see k purple ones?

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