# Linearity of Expectation - Proof

**Linearity of Expectation**

**For any two random variables $X$ and $Y$:**

$$\mathbb{E}[\boldsymbol{X}+\boldsymbol{Y}] = \mathbb{E}[\boldsymbol{X}] + \mathbb{E}[\boldsymbol{Y}]$$

**Note: $X$ and $Y$ do not have to be independent**

Proof:

$$\begin{aligned}\mathbb{E}[X+Y] &= \Sigma_{\omega\in\Omega}\mathbb{P}(\omega)\big(X(\omega)+Y(\omega)\big) \\ &= \Sigma_{\omega\in\Omega}\mathbb{P}(\omega)X(\omega)+\mathbb{P}(\omega)Y(\omega) \\ &= \Sigma_{\omega\in\Omega}\mathbb{P}(\omega)X(\omega)+\Sigma_{\omega\in\Omega}\mathbb{P}(\omega)Y(\omega) \\ &= \mathbb{E}[X]+\mathbb{E}[Y]\end{aligned}$$

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# Repeated Coin Tosses (Again)

The probability of flipping a head is $p$ and we want to find the total number of heads flipped when we flip the coin $n$ times?

Let $X$ be the total number of heads

What indicators can we define? What 'Booleans' have enough information to combine (add) and solve the problem?

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## Pairs with the same birthday

In a class of $m$ students, on average how many pairs of people have the same birthday?

Decompose:

LOE:

Conquer:

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## Rotating the table

$n$ people are sitting around a circular table. There is a name tag in each place. Nobody is sitting in front of their own name tag.

Rotate the table by a random number $k$ of positions between 1 and n-1 (equally likely)

Let $X$ be the number of people that end up in front of their own name tag. **Find $\mathbb{E}[X]$.**

Decompose:

What $X_i$ can we define that have the needed information?

LOE:

Conquer:

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