Conditional Probability

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For an event B, with $\mathbb{P}(B) > 0$,
the "Probability of A conditioned on $\boldsymbol{B}$" is

$$\mathbb{P}(\boldsymbol{A}|\boldsymbol{B}) = \frac{\mathbb{P}(\boldsymbol{A} \cap \boldsymbol{B})}{\mathbb{P}(\boldsymbol{B})}$$

Just like with the formal definition of probability, this is pretty abstract.
It does accurately reflect what happens in the real world.

If $\mathbb{P}(B) = 0$, we can't condition on it (it can't happen! There's no point in defining probabilities where we know B has not happened) – $\mathbb{P}(A|B)$ is **undefined** when $\mathbb{P}(B) = 0$.

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Willy Wonka

Willy Wonka has placed golden tickets on 0.1% of his Wonka Bars.

- If the bar you weigh **does** have a golden ticket, the scale will alert you 99.9% of the time.
- If the bar you weigh does not have a golden ticket, the scale will (falsely) alert you only 1% of the time.

You pick up a bar and it alerts, what is the probability you have a golden ticket?

Which of these is closest to the right answer?

A. 0.1%
B. 10%
C. 50%
D. 90%
E. 99%
F. 99.9%

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LTP and Bayes

Law of Total Probability

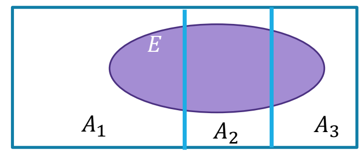
Let $A_1, A_2, \ldots, A_k$ be a **partition** of Ω.
For any event E,

$$\mathbb{P}(\boldsymbol{E}) = \sum_{\textbf{all } i} \mathbb{P}(\boldsymbol{E}|\boldsymbol{A_i})\mathbb{P}(\boldsymbol{A_i})$$

Bayes' Rule

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$$

Ω, split into partition A_1, A_2, A_3 with event E inside.



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A contrived example

You have three red marbles and one blue marble in your left pocket, and one red marble and two blue marbles in your right pocket.

You will flip a fair coin; if it's heads, you'll draw a marble (uniformly) from your left pocket, if it's tails, you'll draw a marble (uniformly) from your right pocket.

Let B be you draw a blue marble. Let T be the coin is tails.

What is $\mathbb{P}(B|T)$ what is $\mathbb{P}(T|B)$?

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