Here Early?

Here for CSE 312?

Welcome! You’re early!

Want a copy of these slides to take notes?

- You can download them from the webpage cse.uw.edu/312

Introduction and Counting

CSE 312 Summer 25
Lecture 1

Staff

Instructor: Anna Kuznetsova

Graduate Student in Computer Science at UW

Email: avk5@uw.edu

TAs

Caleb Hsu
Kieran Rullman
Jewoo Park
Krisha Khandelwal
Thakrit (Jai) Wongcharoensangsiri

Logistics – Lectures

There is one lecture, MWF

12:00 – 1:00pm in DEM 104

Lectures will be recorded, with recordings posted when possible.

Logistics – Sections

Sections meet on Thursdays (starting this week)

Please go to your assigned section.

Both sections are at 12pm in ECE.

Sections will **not** be recorded – we want you to be able to ask questions and give feedback without worrying about being recorded.

Handouts and solutions will be posted.

Your participation, whether in person or async will be recorded.

Materials

When in doubt, it’s on the webpage:

Full URL: https://courses.cs.washington.edu/courses/cse312/25su/

Short URL: http://cse.uw.edu/312

Work

Approximately 7 Homeworks (20%)
Mostly written problems, but a few programming questions.

Concept Checks (10%)
Short assignment for each lecture on gradescope; identify misconceptions right away.

Due the morning of the next lecture. First due date Friday.

Section Participation (10%)
Go to section and work on problems there or (if you can't make it) do the corresponding problems on your own and email them to your TA by Sunday at 11:59pm.

7 In-Lecture Quizzes (10%)
Tentative topics are posted on the calendar. First quiz this Friday.

Two quizzes will be dropped to account for sicknesses and other things that come up during the quarter.

Midterm (20%)
In lecture on Wednesday July 23rd at 12pm.

Final exam (30%)
Part 1 in your assigned section Thursday August 21st at 12pm.

Part 2 in lecture, Friday August 22nd at 12pm.

Final is cumulative but will focus on the latter part of the course.

Communication

Ed Discussion board will be our primary means of communication.

Please check frequently.

We'll send announcement emails via Ed.

If you want to contact us:

- Private post on Ed (seen by TAs and Anna, but not other students)
- Email Anna
- Anonymous Feedback form on webpage

Resources

Office Hours

Will start this Friday. Check the calendar for times and locations.

1:1 Office Hours

See course website for more details.

Ed Discussion

Ask general questions in the mega threads.

Ask specific questions as private posts.

Resources page

List of theorems and definitions

Optional Textbook

Academic Integrity

Be sure that you are able to explain and/or re-derive anything that you submit. If we have doubts about whether you did the work on your own, we may ask you to come in and explain your solution to us verbally.

Collaboration

Resources outside of the course

Generative AI (ChatGPT, LLMs, etc.)

What is This Class?

We're going to learn fundamentals of probability theory.

A **beautiful** and *useful* branch of mathematics.

Applications in:
Machine Learning
Natural Language Processing
Cryptography
Error-Correcting Codes
Data Structures
Data Compression
Complexity Theory
Algorithm Design
…

Content

Combinatorics (*fancy* counting)

Permutations, combinations, inclusion-exclusion, pigeonhole principle

Formal definitions for Probability

Probability space, events, conditional probability, independence, expectation, variance

Common patterns in probability

Equations and inequalities, “zoo” of common random variables, tail bounds

Continuous Probability

pdf, cdf, sample distributions, central limit theorem, estimating probabilities

Applications

Across CS, but with some focus on ML.

Themes

Precise mathematical communication

Both reading and writing dense statements.

Probability in the “real world”

A mix of CS applications

And some actual “real life” ones.

Refine your intuition

Most people have some base level feeling of what the chances of some event are.

We’re going to train you to have better gut feelings.

Counting

Why Counting?

Sometimes useful for algorithm analysis.

- The easiest code to write for “find X” is “try checking every spot where X could be”
- “Given an array, find a set of elements that sum to 0”
- “Given an array, find a set of 2 elements that sum to 0”
- Gut check of “we can ‘brute force’ this or we can’t” is super useful.

A building block toward probability theory

“What are the chances” is usually calculated by

$$= \frac{\text{how many ways can I succeed}}{\text{how many ways can I succeed} + \text{how many ways can I fail}}$$

Remember sets?

A set is an **unordered** list of elements, ignoring repeats.

$\{1,2,3\}$ is a set. It’s the same set as $\{2,1,3\}$.

$\{1,1,2,3\}$ is a very confusing way of writing the set $\{1,2,3\}$.

The **cardinality** of a set is the number of elements in it.

$\{1,2,3\}$ has cardinality 3

$|\{1,2,3\}| = 3$.

Counting Rules

How many options do I have for dinner in Pelican Town?

I could go to Pierre's General Store where there are 3 meals I choose from, or I could go to The Stardrop Saloon where there are 5 meals I choose from (and none of them are the same between the two).

How many total choices?

$3 + 5 = 8$

Sum Rule

If you are choosing one thing between n options in one group and m in another group with no overlap, the total number of options is: $n + m$.

Counting Rules

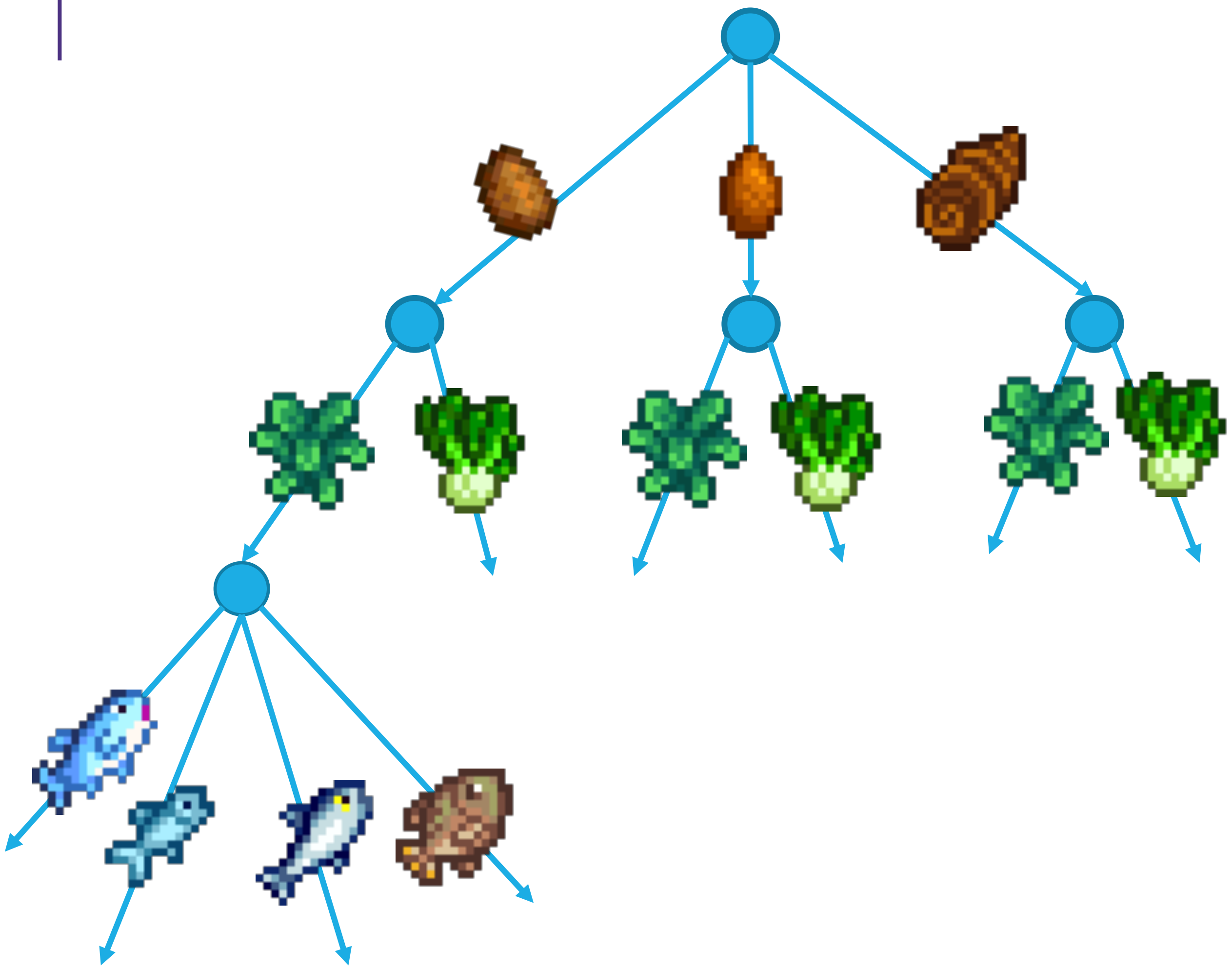
I'm still hungry…

I decide to cook a meal myself. My meals are always:

- One of three types of root vegetable (potato, yam, or taro root).
- One of two leafy vegetables (kale or bok choy)
- One of four fish (tuna, sardine, salmon, or tilapia)

How many meals can I make?

Meals



Step 1: choose one of the three root vegetables.

Step 2: regardless of step 1, choose one of the two leafy vegetables.

Step 3: regardless of steps 1 and 2, choose one of the four fish.

$$3 \cdot 2 \cdot 4 = 24.$$

Counting Rules

Sum Rule

If you are choosing one thing between n options in one group and m in another group with no overlap, the total number of options is: $n + m$.

Product Rule

If you have a sequential process, where step 1 has n_1 options, step 2 has n_2 options,…,step k has n_k options, and you choose one from each step, the total number of possibilities is $n_1 \cdot n_2 \cdots n_k$

Applications of the product rule

Remember Cartesian products?

$S \times T = \{(x, y): x \in S, y \in T\}$

$\{1,2\} \times \{a, b, c\} = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$

How big is $S \times T$? (i.e. what is $|S \times T|$?)

Step 1: choose the element from S.

Step 2: choose the element from T.

Total options: $|S| \cdot |T|$

Power Sets

$\mathcal{P}(S) = \{X : X \subseteq S\}$

$\mathcal{P}(\{1,2,3\}) = \{\ \emptyset,\ \{1\},\{2\},\{3\},\ \{1,2\},\{1,3\},\{2,3\},\ \{1,2,3\}\ \}$

How many subsets are there of S, i.e. what is $|\mathcal{P}(S)|$?

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How many subsets are there of S, i.e. what is $|\mathcal{P}(S)|$?

If $S = \{e_1, e_2, \dots, e_{|S|}\}$

Step 1: is e_1 in the subset?

Step 2: is e_2 in the subset?

…

Step $|S|$: is $e_{|S|}$ in the subset?

$2 \cdot 2 \cdots 2$, |S| times, i.e., $2^{|S|}$.

Farming Outfits

The farmer has three hats (cowboy, straw, mushroom) 

Three shirts (classic overalls, night sky, light blue) 

And three pairs of pants (farmer, dinosaur, relaxed fit) 

How many outfits are there (consisting of one hat, shirt, and pair of pants) if

the classic overalls shirt cannot be worn with dinosaur pants, 

the night sky shirt cannot be worn with relaxed fit pants, 

and the light blue shirt cannot be worn with farmer pants. 

Farming Outfits

Step 1: 3 choices for hats.

Step 2: 3 choices for shirts

Step 3:…

Farming Outfits

Step 1: 3 choices for hats.

Step 2: 3 choices for shirts.

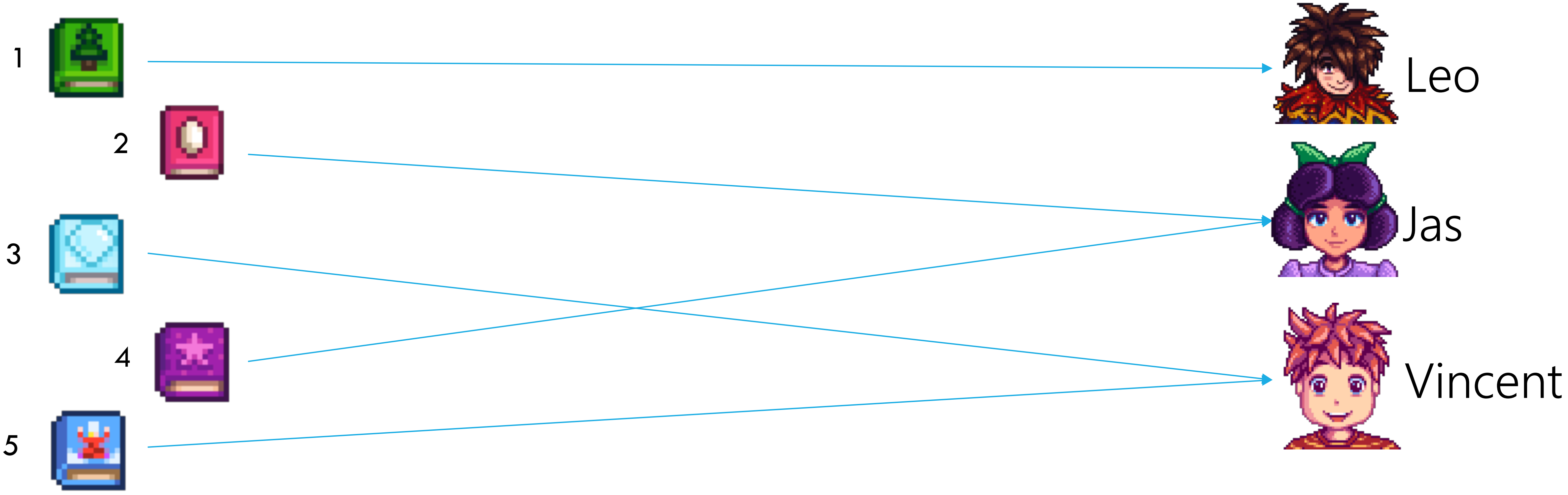
Step 3: Regardless of which shirt we choose, we have 2 options for pants (even though there are three options overall).

$3 \cdot 3 \cdot 2 = 18.$

Assigning Books

We have 5 books to split to 3 people (Leo, Jas, and Vincent)

Every book goes to exactly one person, but each person could end up with no books (or all of them, or something in between).



Assigning Books

We have 5 books to split to 3 people (Leo, Jas, and Vincent)

Every book goes to exactly one person, but each person could end up with no books (or all of them, or something in between).

Attempt 1: We're choosing subsets!

Leo could get any of the $2^5 = 32$ subsets of the books.

Jas could get any of the $2^5 = 32$ subsets of the books.

Vincent could get any of the $2^5 = 32$ subsets of the books.

Total is product of those three steps $32 \cdot 32 \cdot 32 = 32768$

Activity

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We overcounted!

If Leo gets $\{1,2\}$, Jas can't get any subset, she can only get a subset of $\{3,4,5\}$. And Vincent's subset is just whatever is leftover after Leo and Jas get theirs…

Fixing All The Books

You **could**

List out all the options for Leo.

For each of those (separately), list all the possible options for Jas and Vincent.

Use the Summation rule to combine.

~OR~ you could come at the problem from a different angle.

Fixing All the Books

Instead of figuring out which books Leo gets, choose book by book which person they go to.

Step 1: Book 1 has 3 options (Leo, Jas, or Vincent).

Step 2: Book 2 has 3 options (Leo, Jas, or Vincent).

…

Step 5: Book 5 has 3 options.

Total: 3^5.

More sequence practice

How many length 3 sequences are there consisting of distinct elements of $\{1,2,3\}$.

Pause

How many length 3 sequences are there consisting of distinct elements of $\{1,2,3\}$.

Questions in combinatorics and probability are often dense. A single word can totally change the answer. Does order matter or not? Are repeats allowed or not? What makes two things “count the same” or “count as different”?

Let’s look for some keywords

More sequence practice

How many length 3 sequences are there consisting of distinct elements of {1,2,3}.

Sequences implies that order matters – (1,2,3) and (2,1,3) are different.
Distinct implies that you can't repeat elements (1,2,1) doesn't count.

{1,2,3} is our "universe" – our set of allowed elements.

More sequence practice

How many length 3 sequences are there consisting of distinct elements of $\{1,2,3\}$.

Step 1: 3 options for the first element.

Step 2: 2 (remaining) options for the second element.

Step 3: 1 (remaining) option for the third element.

$3 \cdot 2 \cdot 1$

Factorial

That formula shows up a lot.

The number of ways to “permute” (i.e. “reorder”, i.e. “list without repeats”) n elements is "n factorial”

n factorial

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 1$$

We only define $n!$ for natural numbers n.

As a convention, we define: $0! = 1$.

More Practice

Strings

How many strings of length 5 are there over the alphabet $\{A, B, C, \ldots, Z\}$? (repeated characters allowed)

How many binary strings of length n are there?

Strings

How many strings of length 5 are there over the alphabet $\{A, B, C, \dots, Z\}$? (repeated characters allowed)

26^5

How many binary strings of length n are there?

2^n