Homework 1: Counting

For each problem, remember you must briefly explain/justify how you obtained your answer, as correct answers without an explanation will not receive full credit. Moreover, in the event of an incorrect answer, we can still try to give you partial credit based on the explanation you provide.

In general, your goal in an explanation is to write enough that a student from class who has attended lecture, but not thought about the problem yet, could understand your approach, verify your reasoning, and believe your answer is correct. While we do not usually need to see arithmetic, you must include enough work that in principle one could rederive your answer with only a scientific calculator.

Unless a problem states otherwise, you should leave your answer in terms of factorials, combinations, etc., for instance 26^7 or $26!/7!$ or $26 \cdot \binom{26}{7}$ are all good forms for final answers.

Remember that you must "select pages" to identify which problem is on which page as part of your gradescope upload.

Submission: There are four gradescope boxes for this homework:

- You must upload a **pdf** of your written solutions to Gradescope under "HW 1 [Written]". (Instructions as to how to upload your solutions to gradescope are on the course web page.) The use of *LaTeX* is *highly recommended*. (Note that if you want to hand-write your solutions, you'll need to scan them. We will not grade work that we are unable to read, so please write neatly if you handwrite.)
- Your code will be submitted under "HW 1 [Coding]" as files called `cse312_pset1_vars.py`, `cse312_pset1_loops.py`, `cse312_pset1_lists.py`, `cse312_pset1_functions.py`, and `cse312_pset1_classes.py`..
- You will do an "online assignment" on gradescope to complete problem 0.
- The feedback problem (problem 8) has a separate box.

Due Date: This assignment is due at 11:59 PM Wednesday July 2nd.

Academic Integrity: Please read the full academic integrity policy. If you work with others (and you should!), you must still write up your solution independently and name all of your collaborators in the separate question on gradescope.

0. Mathematical Background [10 points]

Complete the mathematical background problems located in Problem 0 on gradescope. This assignment will operate much like a concept check—an explanation will appear when you get a problem right, and you may resubmit until you get all problems correct.

The material in those problems are things we expect you learned in prior courses (especially calculus courses). We'll do similar computations as part of bigger problems later in the quarter. If there are any topics you aren't comfortable with, you should carve some time out to review over the next few weeks.

1. Syllabus [5 points]

Read the Syllabus, especially the academic integrity policy, late policy, and the exam schedule. If you have questions, please ask them on Ed!

On your homework submission write exactly "I have read the syllabus, and agree to follow the academic integrity policies." for full credit.

2. Bands [15 points]

Five guitar players, four bass players and three drummers show up to form a band.

(a) For the upcoming competition, bands will consist of 4 people. How many ways are there choose which 4 people will be in the band? (for this part, you should just account for who is playing and who is not). Note: bands can be comprised of any 4 instruments

(b) There will be 4 solos throughout the performance. How many ways are there to assign the 4 solos by selecting from the 12 possible players (for this part, account for both who is in the band and which of the 4 solos they are playing. Each solo occurs at a different time)? Note: bands can be comprised of any 4 instruments

(c) After much auditioning, we've realized we need a drummer. How many ways are there to choose which 4 players will be in the band if at least one of these players must be a drummer (for this part, we select only who are in the band and not their positions like part (a))?

3. Mining [20 points]

Diamonds can best be found at an altitude of 11. Suppose you are mining at this altitude starting from a position (0,11,0) and have received the following information. Note: treat each part as an independent question. For instance, the information provided in part (b) does not apply to that in part (c) or (d).

(a) You've deduced there exists a diamond at (125,11,150). How many paths are there from your starting position to this diamond if every step **increments** one of the x or z coordinates and leaves the other unchanged (Hint: the coordinates correspond to (x,y,z))?

(b) A friend told you there is an emerald at (80,11,80). How many paths are there from your starting position to the diamond at position (125,11,150) if every step increments one of the x or z coordinates and leaves the other unchanged AND you want the path to contact/go through the emerald.

(c) There are pools of lava at (30,11,40) and (70,11,80). How many paths are there from point (0,11,0) to (125,11,150) if every step increments one x or z coordinate and leaves the other unchanged and the path **cannot** go through either lava pool? (Hint: Try inclusion-exclusion.)

(d) You are done mining and want to return to your home at (20,40,300). How many paths from your starting position (0,0,0) to home are there if every step increments one coordinate and leaves the other two unchanged? Like the other parts

4. 3 Suited Deck [10 points]

Your friends have just created a new card game that involves a non-standard deck of cards. The deck has 3 suits, each of which has 14 ranks (therefore 42 total cards). How many 4 card hands are there, with at least one card from each suit.

5. Sitting around [15 points]

Abigail (A), Birdie (B), Clint (C), Demetrius (D), Emily (E), Fizz (F), and Gunther (G) are sitting in a row of nine seats (Note: there are only seven people). Demetrius and Gunther are arguing, so they cannot sit next to each other. Clint and Emily are dating, so they must sit next to each other. Abigail, Birdie, and Fizz are best friends, so they also want to sit next to each other, but Birdie must be in the middle of Abigail and Fizz (with no spaces between the three). Our goal is to figure out how many ways they can sit in a row. Build up to the answer by answering the following questions: In how many ways can they sit in a row? (Hint: We will start by grouping Abigail, Birdie, and Fizz, as well as Clint and Emily. Then we will work on placing Demetrius and Gunther). Also, you will find the problem easier if you just call them, A,B,C,D, E, F, and G, as we do from now on.

(a) How many ways are there to place the 7 people into the 9 chairs if ABF must sit together in that order and CE must sit together in that order (This is not unlike the rearrangements of RHUBARB that we discussed/will discuss in lecture (where the empty seats are like the two R/Bs))?

(b) How many ways are there to place the 7 people into the 9 chairs if ABF must sit together but A and F can swap positions and CE must sit together in either order?

(c) How many ways are there to place the 7 people into the 9 chairs if ABF must sit together (but A and F can swap positions), CE must sit together in either order and DG must sit together in either order?

(d) How many ways are there to place the 7 people into the 9 chairs if ABF must sit together (but A and F can swap positions), CE must sit together in either order and D and G must not sit next to each other?

6. Binomial Theorem applications [15 points]

For part (a) of this question (as with many others for 312), you could find the numerical answer in a few seconds by asking wolfram alpha (in this case, by asking it to expand the polynomial). We have learning goals associated with this problem that mean we want you to practice solving this problem by hand even though you could easily answer it with computational power.

Remember that you must give an explanation of an answer such that another student would understand the **principles** that go into solving the problem, and such that they could find the answer with a simple calculator (that doesn't have an "expand a polynomial" operation)

You may find it beneficial to verify your answer using WolframAlpha, but you may not use the "show steps" option on WolframAlpha or any similar tool.

(a) What is the coefficient of x^5y^{10} in the expansion of $(2x - y^2)^{10}$?

(b) Use the binomial theorem to prove that

$$\sum_{i=0}^{100} \binom{100}{i} (-4)^{100-i} = 3^{100}$$

7. Coding and Reflection [15 points]

(a) Read the HW1 Python Tutorial and Coding Exercises lesson on Edstem and follow the directions to complete the 5 coding exercises. Then submit all required files to HW1 [Coding] on Gradescope. You may resubmit to gradescope as many times as you like. We do not have any hidden tests for this assignment; whatever score you see on gradescope on your last submission will be your final score.

(b) Read the Edstem lesson on Python's numpy library and write the reflection described below (be sure to finish part (a) before moving to this part). You do **not** need to complete the *coding* exercises for this part. The coding exercise that is there is optional, and intended only for practice if you want it. The goal of this part is to see some more advanced python features that will definitely be useful in the future, but you won't need to use frequently for this course.
After reading the lesson, write a reflection on what you felt was the most confusing numpy function and/or class to you and why. If nothing is confusing, explain which function and/or class is the most interesting to you. We will grade based on completion and effort (since this response can't be "correct" or "incorrect"). We expect most responses will be 2-4 sentences. **Include your reflections in the pdf you upload to gradescope with the other problems on this assignment.**

8. Feedback + Collaboration [1 point]

Answer these questions on the separate Gradescope box for this question.

Please keep track of how much time you spend on this homework and answer the following questions. This can help us calibrate future assignments and future iterations of the course, and can help you identify which areas are most challenging for you.

- Which students did you collaborate with for this homework?
- Is the work that you are submitting your own and does not violate the academic integrity policy outlined in the syllabus?
- How many hours did you spend working on this assignment (excluding any extra credit questions, if applicable)? Report your estimate to the nearest hour.
- Which problem did you spend the most time on?
- Any other feedback for us?