Different dice

Roll two fair dice independently. Let U be the minimum of the two rolls and V be the maximum

Are U and V independent?

Write the joint distribution in the table

What's $p_U(z)$? (the marginal for U)

$p_{U,V}$	U=1	U=2	U=3	U=4
V=1				
V=2				
V=3				
V=4				

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Expectation of a function of two RVs

What's $\mathbb{E}[UV]$ for U, V from the last slide?

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Conditional Expectation

Waaaaaay back when, we said conditioning on an event creates a new probability space, with all the laws holding.

So we can define things like "conditional expectations" which is the expectation of a random variable in that new probability space.

$$\mathbb{E}[\boldsymbol{X}|\boldsymbol{E}] = \sum_{\boldsymbol{x}\in\boldsymbol{\Omega}} \boldsymbol{x} \cdot \mathbb{P}(\boldsymbol{X} = \boldsymbol{x}|\boldsymbol{E})$$

$$\mathbb{E}[\boldsymbol{X}|\boldsymbol{Y} = \boldsymbol{y}] = \sum_{\boldsymbol{x}\in\boldsymbol{\Omega}_{\boldsymbol{X}}} \boldsymbol{x} \cdot \mathbb{P}(\boldsymbol{X} = \boldsymbol{x}|\boldsymbol{Y} = \boldsymbol{y})$$

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Covariance

We sometimes want to measure how "intertwined" X and Y are – how much knowing about one of them will affect the other.

If X turns out "big" how likely is it that Y will be "big" how much do they "vary together"

Covariance

$$\mathbf{Cov}(\mathbf{X}, \mathbf{Y}) = \mathbb{E}[(\boldsymbol{X} - \mathbb{E}[\boldsymbol{X}])(\boldsymbol{Y} - \mathbb{E}[\boldsymbol{Y}])] = \mathbb{E}[\boldsymbol{XY}] - \mathbb{E}[\boldsymbol{X}]\mathbb{E}[\boldsymbol{Y}]$$

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