Expectation

$\ln\left(\mathcal{L}\left(x_i;\theta_\mu,\theta_{\sigma^2}\right)\right)=\sum_{i=1}^{n}\ln\left(\frac{1}{\sqrt{\theta_{\sigma^2}2\pi}}\right)-\frac{1}{2}\cdot\frac{\left(x_i-\theta_\mu\right)^2}{\theta_{\sigma^2}}$

$\frac{\partial}{\partial\theta_\mu}\ln(\mathcal{L})=$

Setting equal to $\mathbf{0}$ and solving

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Variance

$\ln\left(\mathcal{L}\left(x_i;\theta_\mu,\theta_{\sigma^2}\right)\right)=\sum_{i=1}^{n}\ln\left(\frac{1}{\sqrt{\theta_{\sigma^2}2\pi}}\right)-\frac{1}{2}\cdot\frac{\left(x_i-\theta_\mu\right)^2}{\theta_{\sigma^2}}$

Take the partial derivative with respect to θ_{σ^2}. It'll be easier if you apply some log and exponent rules first.

$\log(x^y)=y\cdot\log(x)$.

$\log(ab)=\log(a)+\log(b)$.

$\frac{1}{\sqrt{a}}=a^{-1/2}$

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Are our MLEs biased?

Our estimate for the coin-flips (if we generalized a bit) would be $\frac{\text{num heads}}{\text{total flips}}$

Is this biased or unbiased?

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Confidence for MLEs

We said our MLE for "probability of heads on a flip" is $\hat{p} = \frac{\text{num heads}}{\text{num flips}}$

And $\mathbb{E}[\hat{p}] = p$. (where p is the true probability of heads).

But how close is it to the true value? What if on-average it's correct, but it's often very far away.

If only we had a tool…one that would describe the probability of being far from your expectation…

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