

Let's calculate an expectation

Let X be a uniform random number between a and b .

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} z \cdot f_X(z) \, dz$$

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Continuous Uniform Distribution

$X \sim \text{Unif}(a, b)$ (uniform real number between a and b)

$$\text{PDF: } f_X(k) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq k \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$\text{CDF: } F_X(k) = \begin{cases} 0 & \text{if } k < a \\ \frac{k-a}{b-a} & \text{if } a \leq k \leq b \\ 1 & \text{if } k \geq b \end{cases}$$

$$\mathbb{E}[X] = \frac{a+b}{2}$$

$$\text{Var}(X) = \frac{(b-a)^2}{12}$$

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Continuous Zoo

$$X \sim \text{Unif}(a, b)$$

$$f_X(k) = \frac{1}{b-a}$$

$$\mathbb{E}[X] = \frac{a+b}{2}$$

$$\text{Var}(X) = \frac{(b-a)^2}{12}$$

$$X \sim \text{Exp}(\lambda)$$

$$f_X(k) = \lambda e^{-\lambda k} \text{ for } k \geq 0$$

$$\mathbb{E}[X] = \frac{1}{\lambda}$$

$$\text{Var}(X) = \frac{1}{\lambda^2}$$

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

$$f_X(k) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$\mathbb{E}[X] = \mu$$

$$\text{Var}(X) = \sigma^2$$

It's a smaller zoo, but it's just as much fun!

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Comparing Discrete and Continuous

	Discrete Random Variables	Continuous Random Variables
Probability 0	Equivalent to impossible	All impossible events have probability 0, but not conversely.
Relative Chances	PMF: $p_X(k) = \mathbb{P}(X = k)$	PDF $f_X(k)$ gives chances relative to $f_X(k')$
Events	Sum over PMF to get probability	Integrate PDF to get probability
Convert from CDF to P(M/D)F	Sum up PMF to get CDF. Look for "breakpoints" in CDF to get PMF.	Integrate PDF to get CDF. Differentiate CDF to get PDF.
$\mathbb{E}[X]$	$\sum_{\omega} X(\omega) \cdot p_X(\omega)$	$\int_{-\infty}^{\infty} z \cdot f_X(z) \, dz$
$\mathbb{E}[g(X)]$	$\sum_{\omega} g(X(\omega)) \cdot p_X(\omega)$	$\int_{-\infty}^{\infty} g(z) \cdot f_X(z) \, dz$
$\text{Var}(X)$	$\mathbb{E}[X^2] - (\mathbb{E}[X])^2$	$\mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \int_{-\infty}^{\infty} (z - \mathbb{E}[X])^2 f_X(z) \, dz$

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