

The probability density function

For Continuous random variables, the analogous object is the “probability density function” we write $f_X(k)$ instead of $p_X(k)$

Idea: Make it “work right” for **events** since single outcomes don’t make sense.

$$\mathbb{P}(0 \leq X \leq 1) = 1$$

integrating is analogous to sum.

$$\mathbb{P}(X \text{ is negative}) = 0$$

$$\mathbb{P}(.4 \leq X \leq .5) = .1$$

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Comparing Discrete and Continuous

	Discrete Random Variables	Continuous Random Variables
Probability 0	Equivalent to impossible	All impossible events have probability 0, but not conversely.
Relative Chances	PMF: $p_X(k) = \mathbb{P}(X = k)$	PDF $f_X(k)$ gives chances relative to $f_X(k')$
Events	Sum over PMF to get probability	Integrate PDF to get probability
Convert from CDF to PMF	Sum up PMF to get CDF. Look for “breakpoints” in CDF to get PMF.	Integrate PDF to get CDF. Differentiate CDF to get PDF.
$\mathbb{E}[X]$	$\sum_{\omega} X(\omega) \cdot p_X(\omega)$	$\int_{-\infty}^{\infty} z \cdot f_X(z) \, dz$
$\mathbb{E}[g(X)]$	$\sum_{\omega} g(X(\omega)) \cdot p_X(\omega)$	$\int_{-\infty}^{\infty} g(z) \cdot f_X(z) \, dz$
$\text{Var}(X)$	$\mathbb{E}[X^2] - (\mathbb{E}[X])^2$	$\mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \int_{-\infty}^{\infty} (z - \mathbb{E}[X])^2 f_X(z) \, dz$

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Let's calculate an expectation

Let X be a uniform random number between a and b .

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} z \cdot f_X(z) \, dz$$

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Continuous Uniform Distribution

$X \sim \text{Unif}(a, b)$ (uniform real number between a and b)

$$\text{PDF: } f_X(k) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq k \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$\text{CDF: } F_X(k) = \begin{cases} 0 & \text{if } k < a \\ \frac{k-a}{b-a} & \text{if } a \leq k \leq b \\ 1 & \text{if } k \geq b \end{cases}$$

$$\mathbb{E}[X] = \frac{a+b}{2}$$

$$\text{Var}(X) = \frac{(b-a)^2}{12}$$

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