Another Example

Suppose you shuffle a deck of cards so any arrangement is equally likely. What is the probability that the top two cards have the same value?

Sample Space

Probability Measure

Event

Probability

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Uniform Probability Space

The most common probability measure is the **uniform** probability measure. In the uniform measure, for every event E

$\mathbb{P}(E) = \frac{|E|}{|\Omega|}$.

Let your sample space be all possible outcomes of a sequence of 100 coin tosses. Assign the uniform measure to this sample space. What is the probability of the event "there are exactly 50 heads?"

A. $\binom{100}{50}/2^{100}$

B. $1/101$

C. $1/2$

D. $1/2^{50}$

E. There is not enough information in this problem.

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Probable Fruit

The fruit store sells apples, bananas, and oranges.
Robbie will buy a bag of 10 fruits (order doesn't matter) to bring to lecture, uniformly at random among all possible bags that contain at least one of each fruit type.

You and your friend are first in line to take fruit, and will take an apple each if it's available---what is the probability you both get an apple?

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Conditional Probability

Conditional Probability

For an event B, with $\mathbb{P}(B) > 0$,
the "Probability of A conditioned on B" is

$$\mathbb{P}(\boldsymbol{A}|\boldsymbol{B}) = \frac{\mathbb{P}(\boldsymbol{A} \cap \boldsymbol{B})}{\mathbb{P}(\boldsymbol{B})}$$

Just like with the formal definition of probability, this is pretty abstract.
It does accurately reflect what happens in the real world.

If $\mathbb{P}(B) = 0$, we can't condition on it (it can't happen! There's no point in defining probabilities where we know B has not happened) – $\mathbb{P}(A|B)$ is **undefined** when $\mathbb{P}(B) = 0$.

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