Overcounting

How many anagrams are there of GODOGGY?

7

Pascal's Rule: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

$$\begin{aligned}
\binom{n-1}{k-1} + \binom{n-1}{k} &= \frac{(n-1)!}{(k-1)!(n-1-[k-1])!} + \frac{(n-1)!}{k!(n-1-k)!} && \text{definition of combination} \\
&= \frac{(n-1)!}{(k-1)!(n-k)!} + \frac{(n-1)!}{k!(n-k-1)!} && \text{subtraction} \\
&= \frac{[(n-1)!k!(n-k-1)!] + [(n-1)!(k-1)!(n-k)!]}{k!(k-1)!(n-k)!(n-k-1)!} && \text{Find a common denominator} \\
&= \frac{(n-1)!(k-1)!(n-k-1)![k+(n-k)]}{k!(k-1)!(n-k)!(n-k-1)!} && \text{factor out common terms} \\
&= \frac{(n-1)![k+(n-k)]}{k!(n-k)!} && \text{Cancel } (k-1)!(n-k-1)! \\
&= \frac{(n-1)! \cdot n}{k!(n-k)!} = \frac{n!}{k!(n-k)!} && \text{Algebra} \\
&= \binom{n}{k} && \text{Definition of combination}
\end{aligned}$$

14

Example

How many length 5 strings over the alphabet $\{a, b, c, \ldots, z\}$ contain:

Exactly 2 'a's OR

Exactly 1 'b' OR

No 'x's

For what A, B, C do we want $|A \cup B \cup C|$?

21

Practical tips

Give yourself clear definitions of A, B, C.

Make a table of all the formulas you need before you start actually calculating.

Calculate "size-by-size" and incorporate into the total.

Basic check: If (in an intermediate step) you ever:

1. Get a negative value
2. Get a value greater than the prior max by adding (after all the single sets)
3. Get a value less than the prior min by subtracting (after all the pairwise intersections)

Then something has gone wrong.

29