CSE 312 : Quiz 4 Practice 1 Solutions

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Instructions

- You have twenty minutes to complete this exam.
- You are permitted one piece of 8.5x11 inch paper with handwritten notes (notes are allowed on both sides of the paper). You should also get a provided formula sheet (in section it'll be on different colored paper separate from the exam; if you take the exam with DRS it will be the last page of your exam).
- You may not use a calculator or any other electronic devices during the exam.
- We will be scanning your exams before grading them. Please write legibly, and avoid writing up to the edge of the paper.
- If you run out of room, you may also use the last page for extra space, but tell us where to find your answer if it's not right below the problem.
- Since you don't have a calculator, you are generally free to **not** simplify expressions (though you may if you think it will be helpful).
- In general, you should show us the work you used to get to an answer, and explanations will help us reward partial credit, but we do **not** expect explanations at the level we usually require on homeworks.

Advice

- Writing a few words about where an expression came from is often very helpful for awarding partial credit.
- Remember to take deep breaths.

Question	**Max points**
PDF/CDF	20
CLT	14
Grading Morale	1
Total	**35**

1. CLT

(a) A new bagel shop has opened in UDistrict and everyone is dying to try it. The store gets an average of 3 customers per minute and is open for 10 hours (600 minutes) each day. The number of customers appearing each minute is independent of all other minutes, and follows a Poisson distribution. The store will sell out of bagels if they get more than 2500 customers in a day. Use the CLT to approximate the probability that the store sells out of bagels. (You must use individual minutes as your starting variables).

You SHOULD NOT lookup values in the z-table for this problem. Instead, your solution should be an expression with Φ that can be evaluated using the lookup table (i.e. all inputs to Φ are non-negative) and a calculator (i.e. the input to Φ does not need to be simplified).

Solution:

Let X_i = the number of customers in minute i and X the total number of customers in a day. Then, $X_i \sim \text{Poi}(3)$ with $\mathbb{E}[X] = \text{Var}(X) = 3$ and $X = \sum_{i=1}^{600} X_i$. We are looking for $\Pr(X > 2500)$. Using the CLT, we can approximate X with $W \sim \mathcal{N}(600 \cdot 3, 600 \cdot 3)$.

$$\begin{aligned}
\Pr(X > 2500) &= \Pr(X > 2500.5) && \text{Continuity correction} \\
&\approx \Pr(W > 2500.5) && \text{CLT} \\
&= \Pr\left(\frac{W - 1800}{\sqrt{1800}} > \frac{2500.5 - 1800}{\sqrt{1800}}\right) && \text{Normalization} \\
&= 1 - \Phi\left(\frac{2500.5 - 1800}{\sqrt{1800}}\right)
\end{aligned}$$

2. Concentration

You flip a fair coin (independently) 50 times, and a coin that comes up heads with probability .8 (independently) 50 times. Let X be the number of heads from the fair coin, Y be the number of heads of the unfair coin, and Z be the total number of heads (i.e., $X + Y$).

(a) First, compute $\mathbb{E}[X]$

Solution:

X is a binomial with parameters $50, .5$; $\mathbb{E}[X] = 50 \cdot 0.5 = 25$.

(b) Compute $\text{Var}(X)$

Solution:

The variance of a binomial is $np(1-p)$, which gives $50 \cdot 0.5 \cdot 0.5 = \frac{25}{2}$.

For the next two parts, you may use E to mean $\mathbb{E}[X]$ and V to mean $\text{Var}(X)$.

(c) Use Markov's inequality to bound the probability that $X \geq 40$.

Solution:

Directly applying MArkov, we get $\mathbb{P}(X \geq 40) \leq \frac{\mathbb{E}[X]}{40} = \frac{E}{40} = \frac{25}{40} = \frac{5}{8}$.

(d) Use Chebyshev's Inequality to bound $\mathbb{P}(20 \leq X \leq 30)$. **Solution:**

We reframe he event to look more like Chebyshev: $\mathbb{P}(20 \leq X \leq 30) = \mathbb{P}(20 - 25 \leq X - 25 \leq 30 - 5) = \mathbb{P}(-5 \leq X - \mathbb{E}[X] \leq 5) = \mathbb{P}(|X - \mathbb{E}[X]| \leq 5)$.

We now can apply Chebyshev to get: $\mathbb{P}(20 \leq X \leq 30) \leq \frac{\text{Var}(X)}{5^2} = \frac{25/2}{25} = \frac{1}{2}$.

(e) Can you use the Chernoff bound to bound $\mathbb{P}(Z \geq 90)$? Select ALL that apply.

- ☐ Yes, it would be valid to apply Chernoff, and there is enough information in the problem.
- ☐ No, not enough information is given to compute δ.
- ☐ No, not enough information is given to compute μ.
- ☐ No, Chernoff only applies when the coin has the same probability always.
- ☐ No, Chernoff could only bound $\mathbb{P}(Z \leq 90)$.

Solution:

It's valid to apply Chernoff! There's no requirement that the X_i be identically distributed (just independent of each other, which we have here). The expectation can be computed (it's the sum of two binomials), and since we have a concrete event and μ, we can compute δ.