CSE 312 : Quiz 3 Practice 1 Solutions

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Instructions

- You have twenty minutes to complete this exam.
- You are permitted one piece of 8.5x11 inch paper with handwritten notes (notes are allowed on both sides of the paper). You should also get a provided formula sheet (in section it'll be on different colored paper separate from the exam; if you take the exam with DRS it will be the last page of your exam).
- You may not use a calculator or any other electronic devices during the exam.
- We will be scanning your exams before grading them. Please write legibly, and avoid writing up to the edge of the paper.
- If you run out of room, you may also use the last page for extra space, but tell us where to find your answer if it's not right below the problem.
- Since you don't have a calculator, you are generally free to **not** simplify expressions (though you may if you think it will be helpful).
- In general, you should show us the work you used to get to an answer, and explanations will help us reward partial credit, but we do **not** expect explanations at the level we usually require on homeworks.

Advice

- Writing a few words about where an expression came from is often very helpful for awarding partial credit.
- Remember to take deep breaths.

Question	**Max points**
PDF/CDF	20
CLT	14
Grading Morale	1
Total	**35**

1. PDFs and CDFs

Let $f_X(x) = \begin{cases} cx^2 & \text{for } 1 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$

(a) What value of c makes the PDF valid?

(b) Compute $\mathbb{P}(1 \le X \le 2)$. For this part and all remaining parts, you may leave c as an unknown constant in the computation.

(c) Find the Expected value of X.

(d) Find the Variance of X.

Solution:

(i) For a pdf we know that the area under the code must be 1. thus

We compute:

$$1 = \int_{-\infty}^{\infty} cx^2\,dx.$$

Using the bounds:

$$\int_1^5 cx^2\,dx = \frac{c}{3}x^3|_1^5 = \frac{124c}{3} = 1 \to c = \frac{3}{124}$$

(ii) Evaluating:

$$\int_1^2 \frac{3}{124}x^2\,dx = \frac{3}{124\cdot 3}x^3|_1^2 = \frac{1}{124}2^3 - \frac{1}{124}1^3 = \frac{7}{124} \approx .0564.$$

(iii) Evaluating:

$$E[X] = \int_1^5 x\frac{3}{124}x^2\,dx = \frac{3}{124\cdot 4}x^4|_1^5 = \frac{3}{124\cdot 4}\cdot 5^4 - \frac{3}{124\cdot 4}\cdot 1^4 \approx 3.774...$$

(iv) We start by finding $\mathbb{E}[X^2]$.

$$E[X^2] = \int_1^5 x^2\cdot\frac{3}{124}x^2\,dx = \frac{3}{124\cdot 5}x^5|_1^5 = \frac{3}{124\cdot 5}5^5 - \frac{3}{124\cdot 5}1^5$$

Now using the formula for variance, and plugging in the answer from the last part:

$$\text{Var}(X) = E[X^2] - E[X]^2 = \frac{3}{124\cdot 5}5^5 - \frac{3}{124\cdot 5}1^5 - \left(\frac{3}{124\cdot 4}\cdot 5^4 - \frac{3}{124\cdot 4}\cdot 1^4\right)^2 \approx 0.872.$$

(e) Find the PDF for the following CDF. Be sure to include all cases.

Treat n as an unknown positive integer constant.

$$F_Y(y) = \begin{cases} 0 & \text{for } y < 0 \\ y^n & \text{for } 0 \le y \le 1 \\ 1 & \text{for } y > 1 \end{cases}$$

Solution:

take the derivative of each component of the piecewise

$$f_X(x) = \begin{cases} 0 & \text{otherwise} \\ n \cdot x^{n-1} & 0 \leq x \leq 1 \end{cases}$$