CSE 312 : Practice Quiz 2 Form 2 Solutions

Name:	**NetID:** @uw.edu

Instructions

- You have twenty minutes to complete this exam.
- You are permitted one piece of 8.5x11 inch paper with handwritten notes (notes are allowed on both sides of the paper). You should also get a provided formula sheet (in section it'll be on different colored paper separate from the exam; if you take the exam with DRS it will be the last page of your exam).
- You may not use a calculator or any other electronic devices during the exam.
- We will be scanning your exams before grading them. Please write legibly, and avoid writing up to the edge of the paper.
- If you run out of room, you may also use the last page for extra space, but tell us where to find your answer if it's not right below the problem.
- Since you don't have a calculator, you are generally free to **not** simplify expressions (though you may if you think it will be helpful).
- In general, you should show us the work you used to get to an answer, and explanations will help us reward partial credit, but we do **not** expect explanations at the level we usually require on homeworks.

Advice

- Writing a few words about where an expression came from is often very helpful for awarding partial credit.
- Remember to take deep breaths.

Question	**Max points**
Independence	9
Bayes	16
Total	**25**

There are no problems on this page, go to the next one.

1. Independence

You shuffle a standard deck of cards so that all 52 cards are equally likely to be in any spot. You deal out a 5 card hand (recall that order doesn't matter for hands of cards), then (from the remaining 47 cards) deal out another 5 card hand.

Let A be the event "the first hand contains the King of Hearts."
Let C be the event "the first hand contains the 2 of clubs."
Let B be the event "the second hand contains the King of Hearts."

(a) Are A and B independent? Fill in the appropriate circle **and** do an appropriate calculation to justify your answer. **Be sure to tell us what you're calculating with notation, e.g.,** $\mathbb{P}[X|Y]$**.** [5 points]

◯ A and B are independent.
◯ A and B are **not** independent (i.e., they are dependent).

Solution:

$\mathbb{P}[A] = \frac{\binom{51}{4}}{\binom{52}{5}}$
$\mathbb{P}[B] = \frac{\binom{51}{4}}{\binom{52}{5}}$
$\mathbb{P}[A \cap B] = 0$ (both hands can't have the same card).

Since $\mathbb{P}[A]\mathbb{P}[B] \neq 0 = \mathbb{P}[A \cap B]$, they must be dependent (not independent).

(b) Are B and C independent? You do not need to show us a calculation for this part. [2 points]

◯ B and C are independent.
◯ B and C are **not** independent (i.e., they are dependent).

Solution:

These events are dependent. You can do the exact computation and some careful cancellation to be sure, but it might be easier to notice that if we say the 2 of clubs is in the first hand, we know at least one card that can't stop the king of hearts from being in the second hand, which will produce a (small) dependence:

$\mathbb{P}[B] = \frac{\binom{51}{4}}{\binom{52}{5}}$
$\mathbb{P}[C] = \frac{\binom{51}{4}}{\binom{52}{5}}$
$\mathbb{P}[B \cap C] = \frac{\binom{51}{4} \cdot \binom{47}{4}}{\binom{52}{5} \cdot \binom{47}{5}}$

To fully see these are unequal, cancel a $\binom{51}{4}$ from each numerator and a $\binom{52}{5}$ from each denominator. Starting from: $\mathbb{P}[B]\mathbb{P}[C]$, after cancellations we'd have, $\frac{binom514}{\binom{52}{5}}$. Starting from $\mathbb{P}[B \cap C]$, we'd have $\frac{\binom{47}{4}}{\binom{47}{5}}$. Writing out those combinations, we'll see definitely different expressions.

(c) What is the best description of A, B, C as a group? [2 points]

◯ A, B, C are pairwise independent (but not mutually independent).
◯ A, B, C are mutually independent.
◯ A, B, C are neither mutually nor pairwise independent.

Solution:

Since the answer to (at least) one of the prior parts was dependence, they cannot be either pariwise or mutually independent.

2. Registration Nightmare! [16 points]

Alice will drop the class with probability 0.4. You have enabled MyUW text notifications, so if Alice drops the class, you will get a seat with probability 0.9. If Alice does not drop the class, you can still get in by picking up another seat with probability 0.2.

We define the following events:

- C is the event you get in the class.
- D is the event that Alice drops the class.

Remark: You **DO NOT** need to simplify algebraic expressions (e.g. calculate a final answer). If you need to reference the answer from a previous part, use the variable defined in the header.

(a) What is the probability you get in the class? Call this value a. [4 points]

Solution:

We can solve this using the law of total probability. From the problem statement, we have $\mathbb{P}(D) = 0.4$, $\mathbb{P}(C|D) = 0.9$, and $\mathbb{P}(C|D^C) = 0.2$. By complementation, we have $\mathbb{P}(D^C) = 1 - \mathbb{P}(D) = 1 - (0.4)$. Therefore,

$$\mathbb{P}(C) = \mathbb{P}(C|D)\mathbb{P}(D) + \mathbb{P}(C|D^C)\mathbb{P}(D^C) = (0.9)(0.4) + (0.2)(1 - 0.4)$$

(b) Using the events defined above, what expression represents the probability that Alice drops the class, assuming that you get into the class? [2 points]

$\mathbb{P}$ (__________)

Solution:

$$\mathbb{P}(D|C)$$

(c) Calculate the value for the previous expression. Call this value c. [4 points]

Solution:

From the discussion above, we know $\mathbb{P}(C) = a$, $\mathbb{P}(D) = 0.4$, and $\mathbb{P}(C|D) = 0.9$. Knowing these quantities, we can apply Bayes' rule to get:

$$\mathbb{P}(D|C) = \frac{\mathbb{P}(C|D)\mathbb{P}(D)}{\mathbb{P}(C)} = \frac{(0.9)(0.4)}{a}$$

(d) Using the events defined above, what expression represents the probability that you get into the class **AND** Alice stays in the class? [2 points]

$\mathbb{P}$ (______________)

Solution:

$\mathbb{P}(C \cap D^C)$

(e) Calculate the value for the previous expression. [4 points]

Solution:

One approach:

$$\begin{aligned} \mathbb{P}(C \cap D^C) &= \mathbb{P}(D^C|C)\mathbb{P}(C) && \text{Conditional probability} \\ &= (1 - \mathbb{P}(D|C))\mathbb{P}(C) && \text{Complementation} \\ &= (1 - b)(c) && \text{Using part (a)/(c)} \end{aligned}$$

Another approach:

$$\begin{aligned} \mathbb{P}(C \cap D^C) &= \mathbb{P}(C|D^C)\mathbb{P}(D^C) && \text{Conditional probability} \\ &= \mathbb{P}(C|D^C)(1 - \mathbb{P}(D)) && \text{Complementation} \\ &= (0.2)(1 - 0.4) \end{aligned}$$