

CSE 312 : Practice Quiz 2 (Version 1) Solutions

Name:

NetID:

@uw.edu

Instructions

- You have twenty minutes to complete this exam.
- You are permitted one piece of 8.5x11 inch paper with handwritten notes (notes are allowed on both sides of the paper). You should also get a provided formula sheet (in section it'll be on different colored paper separate from the exam; if you take the exam with DRS it will be the last page of your exam).
- You may not use a calculator or any other electronic devices during the exam.
- We will be scanning your exams before grading them. Please write legibly, and avoid writing up to the edge of the paper.
- If you run out of room, you may also use the last page for extra space, but tell us where to find your answer if it's not right below the problem.
- Since you don't have a calculator, you are generally free to **not** simplify expressions (though you may if you think it will be helpful).
- In general, you should show us the work you used to get to an answer, and explanations will help us reward partial credit, but we do **not** expect explanations at the level we usually require on homeworks.

Advice

- Writing a few words about where an expression came from is often very helpful for awarding partial credit.
- Remember to take deep breaths.

| Question | Max points |
|--------------|------------|
| Independence | 10 |
| Bayes | 15 |
| Total | 25 |

1. Independence

Consider the following silly experiment. You flip a (fair) coin. If it comes up heads, you roll a (fair) 6-sided die. If it comes up tails, you roll a (fair) 4-sided die.

Let H be the event “the coin comes up heads.”

Let S be the event “the die shows a 6.”

Let O be the event “the die shows a 1.”

- (a) Are H and S independent? Fill in the appropriate circle **and** do an appropriate calculation to justify your answer. **Be sure to tell us what you’re calculating with notation, e.g., $\mathbb{P}[X|Y]$.** [4 points]

H and S are independent.

H and S are **not** independent (i.e., they are dependent). **Solution:**

H and S are not independent.

$$\mathbb{P}[H] = 1/2$$

$$\mathbb{P}[S] = \frac{1}{2} \cdot \frac{1}{6} + \frac{1}{2} \cdot 0 = \frac{1}{12}$$

$$\mathbb{P}[H \cap S] = \mathbb{P}[S|H] \mathbb{P}[H] = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$$

$\mathbb{P}[H] \mathbb{P}[S] = \frac{1}{2} \cdot \frac{1}{12} = \frac{1}{24}$, that expression is not equal to $\mathbb{P}[H \cap S]$, so the events are **not** independent (i.e., they are dependent).

- (b) Are H and O independent? You do not need to show us a calculation for this part. [3 points]

H and O are independent.

H and O are **not** independent (i.e., they are dependent).

Solution:

dependent.

$$\mathbb{P}[O] = \frac{1}{2} \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{1}{4} \cdot \mathbb{P}[H \cap O] = \mathbb{P}[O|H] \mathbb{P}[H] = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$$

$$\mathbb{P}[O] \cdot \mathbb{P}[H] = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8} \text{ which is not equal to } \mathbb{P}[H \cap O] \text{ (which is } 1/12).$$

- (c) Are S and O independent? You do not need to show us a calculation for this part. [3 points]

S and O are independent.

S and O are **not** independent (i.e., they are dependent).

Solution:

dependent. There’s a shortcut to see this immediately: S and O are mutually exclusive (so $\mathbb{P}[S \cap O] = 0$), but both are possible, so they cannot be independent. (You can also do the full computation; we already have all the pieces we need from prior parts).

2. (Bayes) Check this box to say you're a human [17 points]

You are designing a CAPTCHA system (a system that checks that users are humans, rather than bots). You know that 80% of the submissions to your system come from bots. If a submission comes from a bot, the bot fails the test 99% of the time; while humans pass the test 75% of the time.

Let H be the event the submission comes from a human, B be the event the submission comes from a bot. Let P be the event the test is passed, and F be the event the test is failed.

- (a) Give the notation and fill in the value for “the probability a test is failed, given that the submission came from a human.” **be sure to fill in both blanks.** [4 points]

$$\mathbb{P}(\text{_____}) = \text{_____}$$

Solution:

$$\mathbb{P}(F|H) = .25$$

- (b) What is the probability a test is failed? [4 points]

Solution:

By Law of Total Probability

$$\mathbb{P}(F) = \mathbb{P}(F|H)\mathbb{P}(H) + \mathbb{P}(F|B)\mathbb{P}(B)$$

Plugging in we get

$$\mathbb{P}(F) = .25 \cdot .2 + .99 \cdot .8$$

- (c) You wish to ban the IP addresses of submitters that you think are bots, but you want to be sure they're really bots. What is the probability a submission came from a bot, given that the test failed. [4 points]
You may use b to represent “the correct answer from part b” **Solution:**

By Bayes Rule

$$\mathbb{P}(B|F) = \frac{\mathbb{P}(F|B) \cdot \mathbb{P}(B)}{\mathbb{P}(F)} = \frac{.99 \cdot .8}{b} = \frac{.99 \cdot .8}{.25 \cdot .2 + .99 \cdot .8}$$

- (d) Suppose that a human gets frustrated after failing a test once, and so when you show the same human a second test, they have only a 50% change of succeeding. What is the probability of a human failing two consecutive tests? [3 points] **Solution:**

$$.25 \cdot .5$$