CSE 312 : Practice Quiz 1 (Form 2) Solutions

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Instructions

- You have twenty minutes to complete this exam.
- You are permitted one piece of 8.5x11 inch paper with handwritten notes (notes are allowed on both sides of the paper). You should also get a provided formula sheet (in section it'll be on different colored paper separate from the exam; if you take the exam with DRS it will be the last page of your exam).
- You may not use a calculator or any other electronic devices during the exam.
- We will be scanning your exams before grading them. Please write legibly, and avoid writing up to the edge of the paper.
- If you run out of room, you may also use the last page for extra space, but tell us where to find your answer if it's not right below the problem.
- Since you don't have a calculator, you are generally free to **not** simplify expressions (though you may if you think it will be helpful).
- In general, you should show us the work you used to get to an answer, and explanations will help us reward partial credit, but we do **not** expect explanations at the level we usually require on homeworks.

Advice

- Writing a few words about where an expression came from is often very helpful for awarding partial credit.
- Remember to take deep breaths.

Question	**Max points**
1. Counting	25
Total	**25**

There are no problems on this page, go to the next one.

1. Passwords [25 points]

You are tasked with designing a secure password system. You will require passwords to meet the following criteria

- Order matters for passwords, and repeating characters **is** allowed.
- A password must be exactly **10 characters long**.
- It can only contain characters from the following three categories:
  - **Alphabet Letters**: 26 lowercase English letters.
  - **Digits**: 10 numeric digits.
  - **Special Characters**: a set of 10 characters such as !, @, etc.

Additionally:

- To be **valid** the password must include:
  - At least **one digit**.
  - At least **one special character**.
- A string of length 10 not meeting these last two criteria is an **invalid password**.

Let:

- $\Omega =$ the set of all passwords of length 10 .
- $D =$ the set of passwords of length 10 containing at least one digit.
- $S =$ the set of passwords of length 10 containing at least one special character.

(a) Using only the sets defined above and set operations, give the **set notation** for each of these sets. An answer for these might look like $\overline{D} \cap S$ [2 points each]

- the set of all valid passwords?

- the set of passwords of length 10 without any digits?

- the set of passwords of length 10 without any special characters?

Solution:

- $D \cap S$
- D^c
- S^c

- the set of invalid passwords as a union of two sets.

Solution:

- $D^c \cup S^c$.
- It is the complement of the set of valid passwords(i.e. $(D \cap S)^c = D^c \cup S^c$).

(b) Give a **formula** for the number of passwords of length 10 (both invalid and valid)? [3 points]

(c) Give a **formula** for the number of passwords of length 10 without any digit? [3 points]

(d) Give a **formula** for the number of passwords of length 10 without any special characters? [3 points]

Solution:

- $|\Omega| = 46^{10}$.
- $|D^c| = 36^{10}$.
- $|S^c| = 36^{10}$.

(e) Compute the size of the set of **invalid** passwords. You may use b, c, d to refer to the correct answers from parts b,c,d. [4 points]

Solution:

First, we use inclusion-exclusion to obtain $|D^c \cup S^c| = |D^c| + |S^c| - |D^c \cap S^c|$. We have computed the first two terms previously. Now, the intersection is simply the set of passwords of length 10 without any digits or special characters. This is equivalent to the set of passwords of length 10 composed only of letters. So, $|D^c \cap S^c| = 26^{10}$. Putting it all together we obtain: $|D^c \cup S^c| = 2 \cdot 36^{10} - 26^{10}$.

(f) Compute the size of the set of **valid** passwords. You may use b, c, d, e to refer to the correct answers of parts b,c,d,e. [4 points] (**Hint**: you should already have most of what you need).

Solution:

We use complementary counting to obtain:

$$|D \cap S| = |\Omega| - |(D \cap S)^c| = |\Omega| - |D^c \cup S^c| = 46^{10} - 2 \cdot 36^{10} + 26^{10}$$