Find a group of 3-5 people to sit with

This is to ensure that we get through all the groups in time when working on problems as groups :)

Section 2

----- More Counting & Probability -----
LOGISTICS

HW 1 due yesterday
(Late deadline Friday(1/12 @ 11:59pm))

Hw 2 is out
(due Wednesday(1/17 @ 11:59pm))

Office Hours
(times/locations listed on the website)
Homework

● Submissions
  ○ LaTeX (highly encouraged)
    ■ overleaf.com
    ■ template and LaTeX guide posted on course website!
  ○ Word Editor that supports mathematical equations
  ○ Handwritten neatly and scanned

● Homework will typically be due on Wednesdays at 11:59pm on Gradescope

● Each assignment can be submitted a max of **48 hours** late

● You have **6 late days total** to use throughout the quarter
  ○ Anything beyond that will result in a deduction on further late assignments
CONTENT REVIEW
NEW TOPICS!

- Binomial Theorem
- Inclusion Exclusion
- Pigeonhole Principle
- Stars and Bars
- Probability Spaces and Uniform Probability
Fun Counting Application:

BINOMIAL THEOREM
Fun Counting Application:

**BINOMIAL THEOREM**

\[(x + y)^n = (x + y) \times (x + y) \times \ldots \times (x + y)\]
Fun Counting Application:

**BINOMIAL THEOREM**

$$(x + y)^n = (x + y) \cdot (x + y) \cdot \ldots \cdot (x + y)$$

Each term in the final sum will choose an $x$ or $y$ from each of the $n$ $(x+y)$’s to get $x^k y^{n-k}$
Fun Counting Application: 

**BINOMIAL THEOREM**

\[(x + y)^n = (x + y) \times (x + y) \times \ldots \times (x + y)\]

Each term in the final sum will choose an \(x\) or \(y\) from each of the \(n\) \((x+y)\) terms to get \(x^k y^{n-k}\)

The coefficient on \(x^k y^{n-k}\) thus will be \(\binom{n}{k}\)
Fun Counting Application: Binomial Theorem

\((x+y)^n = (x+y) \times (x+y) \times \ldots \times (x+y)\)

Each term in the final sum will choose an \(x\) or \(y\) from each of the \(n\) \((x+y)\)s to get \(x^k y^{n-k}\)

The coefficient on \(x^k y^{n-k}\) thus will be \(\binom{n}{k}\)

Is this identical to \(\binom{n}{n-k}\)?
Fun Counting Application: 
BINOMIAL THEOREM

$$(x + y)^n = (x + y) \times (x + y) \times \ldots \times (x + y)$$

Each term in the final sum will choose an $x$ or $y$ from each of the $n$ $(x+y)$s to get $x^k y^{n-k}$

The coefficient on $x^k y^{n-k}$ thus will be $\binom{n}{k}$

Is this identical to $\binom{n}{n-k}$?

Yes! Choosing a set of $k$ out of $n$ things is the same as choosing a set of $n - k$ things to not include
Another counting rule: **inclusion-exclusion**

\[ |A \cup B| \text{ isn't as simple as } |A| + |B| \]
Another counting rule:

**Inclusion-exclusion**

\[|A \cup B| \text{ isn't as simple as } |A| + |B|\]

\[|A \cup B| \text{ is } |A| + |B| - |A \cap B|\]
Another counting rule: **Inclusion-Exclusion**

|A ∪ B| isn’t as simple as |A| + |B|

|A ∪ B| is |A| + |B| − |A ∩ B|

What about |A ∪ B ∪ C|?
Another counting rule: **inclusion-exclusion**

$$|A \cup B| \text{ isn't as simple as } |A| + |B|$$

$$|A \cup B| \text{ is } |A| + |B| - |A \cap B|$$

**What about** $$|A \cup B \cup C|$$?

$$|A \cup B \cup C| \text{ is singles - doubles + triples}$$

$$|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$
Another counting rule: 

**Pigeonhole Principle**
Another counting rule:

**Pigeonhole Principle**

If there are $n$ pigeons with not enough holes for them to stay in ($k$ to be exact), what can we say about at least how many pigeons at least one hole will hold?
Another counting rule:

**Pigeonhole Principle**

If there are $n$ pigeons with not enough holes for them to stay in ($k$ to be exact), what can we say about at least how many pigeons at least one hole will hold?

$\text{ceil}(n / k)$
Another counting rule:

**STARS AND BARS**
Another counting rule:

**STARS AND BARS**

How many ways can you distribute $n$ indistinguishable balls into $k$ distinguishable bins?
Another counting rule: **STARS AND BARS**

How many ways can you distribute \( n \) indistinguishable balls into \( k \) distinguishable bins?

Arrange \( n \) balls and \( k - 1 \) dividers
Another counting rule: **STARS AND BARS**

How many ways can you distribute \( n \) indistinguishable balls into \( k \) distinguishable bins?

Arrange \( n \) balls and \( k - 1 \) dividers

\[
(n + k - 1) \binom{n}{k}
\]
PROBABILITY!
**Probability!**

- **Sample Space:** The set of all possible outcomes of an experiment, denoted $\Omega$ or $S$
- **Event:** Some subset of the sample space, usually a capital letter such as $E \subseteq \Omega$
- **Union:** The union of two events $E$ and $F$ is denoted $E \cup F$
- **Intersection:** The intersection of two events $E$ and $F$ is denoted $E \cap F$ or $EF$
- **Mutually Exclusive:** Events $E$ and $F$ are mutually exclusive iff $E \cap F = \emptyset$
- **Complement:** The complement of an event $E$ is denoted $E^c$ or $\overline{E}$ or $\neg E$, and is equal to $\Omega \setminus E$
- **DeMorgan’s Laws:** $(E \cup F)^c = E^c \cap F^c$ and $(E \cap F)^c = E^c \cup F^c$
Each probability is between 0 and 1 inclusive.

Probabilities add to 1.

If events are *mutually exclusive*,
\[ P(A \cup B \cup C) = P(A) + P(B) + P(C) \]
because there are no intersections.
PROBABILITY!

● Axioms of Probability
  o **Non-negativity**: For any event $E$, $\mathbb{P}(E) \geq 0$
  o **Normalization**: $\mathbb{P}(\Omega) = 1$
  o **Additivity**: If $E$ and $F$ are mutually exclusive events, then $\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F)$

● Corollaries of these axioms
  o **Complementation**: $\mathbb{P}(E) + \mathbb{P}(E^c) = 1$
  o **Monotonicity**: If $E \subseteq F$, $\mathbb{P}(E) \leq \mathbb{P}(F)$
  o **Inclusion-Exclusion**: $\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F)$

● Equally Likely Outcomes: If every outcome in a finite sample space $\Omega$ is equally likely, and $E$ is an event, then $\mathbb{P}(E) = \frac{|E|}{|\Omega|}$
An event is a subset of the sample space 

\[ E \subseteq \Omega \]

If each outcome in the sample space is *equally likely*, the probability of an event is

\[ P(E) = \frac{|E|}{|\Omega|} \]

If the union of a set of mutually exclusive events is equal to the sample sets, those events *partition* the sample space.