

# **SECTION 8**

# ADMINISTRIVIA

- hw 6 due on Monday
- Midterm next Thursday, no section that day.
- Bring cheat sheet, something to write with, husky card.

# **content review**

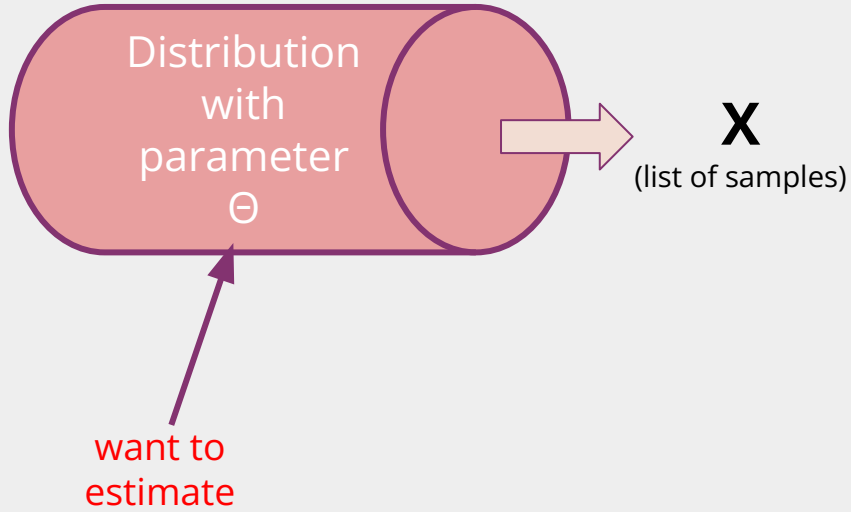
**MLE**

# MLE

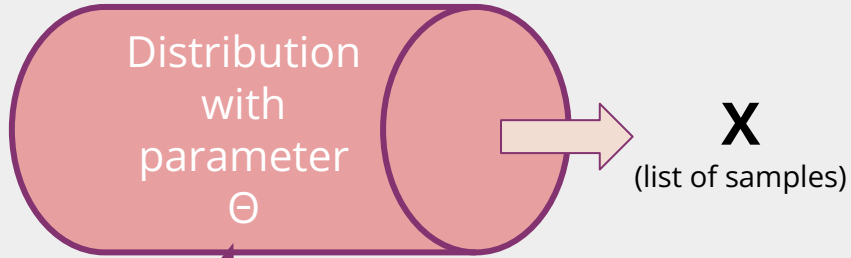
Sometimes, we don't know enough about a particular distribution, and we want to estimate a parameter to that distribution!

***MLE helps us provide an estimate for some parameter(s) to a distribution based on some samples observed from that distribution***

# MLE



# MLE



want to estimate

What value of  $\Theta$  would make the most sense here? What value of  $\Theta$  would have the greatest likelihood of producing this sample?

What value of  $\Theta$  maximizes  $L(X; \Theta)$ ?

# MLE

## Likelihood

Let  $x_1, \dots, x_n$  be iid samples from pmf  $p_X(x; \theta)$  where  $\theta$  are the distribution's parameters. The likelihood function is the probability of seeing the data given the parameters as

$$\mathcal{L}(x_1, \dots, x_n; \theta) = \prod_{i=1}^n p_X(x_i; \theta)$$



# MLE

## Likelihood

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Represents the samples observed - can also just use  $X$  to represent the list of observations

multiplying bc we want the likelihood *all* of these are observed

in the *continuous* case, the PMF is replaced with the PDF

# MLE

## MLE

MLE of  $\theta$  is  $\hat{\theta}$  - the value of  $\theta$  that will **maximize** the likelihood function

$$\hat{\theta} = \arg \max_{\theta} \mathcal{L}(x_1, \dots, x_n; \theta)$$

*We're given that there's a distribution with some unknown parameter(s)  $\theta$ , and there are some observations  $x_1, \dots, x_n$  from this distribution*

### **1. Write the likelihood function**

*even if there are multiple parameters to the distribution, only one likelihood function  
- remember to multiply (not sum) the probability of each of the observations!*

### **2. Take the log of the likelihood function (usually $\ln$ , not $\log$ )**

*we typically want to take natural log ( $\ln$ ) of the likelihood function in order to make finding the derivative much easier (remember  $\ln$  of a product is a sum of the  $\ln$ 's)*

### **3. Take the derivative of the log likelihood function**

*if you're finding an MLE where there are multiple parameters, in this step, take the partial derivative with respect to the parameter you're trying to solve for*

### **4. Set the derivative of the log likelihood function to 0 and solve for $\hat{\theta}$**

*this step where you set the derivative equal to 0 is where you'll want to add the hat on top of the  $\theta$  since at this step, we're solving for the maximum likelihood estimator*

### **5. Verify the estimator is a maximizer via the 2nd derivative test (usually ignore for 312)**

# UNBIASED ESTIMATOR

**Bias:** The bias of an estimator  $\hat{\theta}$  for a **true** parameter  $\theta$  is defined as  $E[\hat{\theta}] - \theta$ . An estimator is unbiased iff  $E[\hat{\theta}] = \theta$ .