Find a group of 3-5 people to sit with
This is to ensure that we get through all the
groups in time when working on problems as
groups :)}
Section 2

More Counting & Probability
LOGISTICS

HW 1 due yesterday
(Late deadline Friday(06/28 @ 11:59pm))

Hw 2 is out
(due Wednesday(07/03 @ 11:59pm))

Office Hours
(times/locations listed on the website)
Homework

- Submissions
  - LaTeX (highly encouraged)
    - overleaf.com
    - template and LaTeX guide posted on course website!
  - Word Editor that supports mathematical equations
  - Handwritten neatly and scanned

- Homework will typically be due on Wednesdays at 11:59pm on Gradescope

- Each assignment can be submitted a max of **48 hours** late
- You have **6 late days total** to use throughout the quarter
  - Anything beyond that will result in a deduction on further late assignments
Content Review
NEW TOPICS!

Binomial Theorem
Inclusion Exclusion
Pigeonhole Principle
Stars and Bars
Probability Spaces and Uniform Probability
Fun Counting Application:

**BINOMIAL THEOREM**
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**BINOMIAL THEOREM**

\[(x + y)^n = (x + y) \times (x + y) \times \ldots \times (x + y)\]
Fun Counting Application:

**Binomial Theorem**

\[(x + y)^n = (x + y) \times (x + y) \times \ldots \times (x + y)\]

Each term in the final sum will choose an $x$ or $y$ from each of the $n$ $(x+y)$’s to get $x^ky^{n-k}$
Fun Counting Application: 

**BINOMIAL THEOREM**

\[(x + y)^n = (x + y) \times (x + y) \times \ldots \times (x + y)\]

Each term in the final sum will choose an x or y from each of the \(n\) \((x+y)\)s to get \(x^k y^{n-k}\)

The coefficient on \(x^k y^{n-k}\) thus will be \(\binom{n}{k}\)
Fun Counting Application: BINOMIAL THEOREM

(x+y)^n = (x+y) \times (x+y) \times \ldots \times (x+y)

Each term in the final sum will choose an x or y from each of the n (x+y)s to get \(x^k y^{n-k}\)

The coefficient on \(x^k y^{n-k}\) thus will be \(\binom{n}{k}\)

Is this identical to \(\binom{n}{n-k}\)?
Fun Counting Application:

**BINOMIAL THEOREM**

\[(x + y)^n = (x + y) \times (x + y) \times \ldots \times (x + y)\]

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The coefficient on \(x^k y^{n-k}\) thus will be \(\binom{n}{k}\)

Is this identical to \(\binom{n}{n-k}\)?

Yes! Choosing a set of \(k\) out of \(n\) things is the same as choosing a set of \(n - k\) things to not include.
Another counting rule: **inclusion-exclusion**

$$|A \cup B| \text{ isn't as simple as } |A| + |B|$$
Another counting rule: **Inclusion-exclusion**

$$|A \cup B| \text{ isn't as simple as } |A| + |B|$$

$$|A \cup B| \text{ is } |A| + |B| - |A \cap B|$$
Another counting rule: **inclusion-exclusion**

|\(A \cup B\)| isn’t as simple as |\(A|+|B\)|

|\(A \cup B\)| is |\(A|+|B|\)−|\(A \cap B\)|

What about |\(A \cup B \cup C\)|?
Another counting rule: **Inclusion-exclusion**

\[ |A \cup B| \text{ isn't as simple as } |A|+|B| \]

\[ |A \cup B| \text{ is } |A|+|B| - |A \cap B| \]

**What about** \( |A \cup B \cup C| ? \)

\[ |A \cup B \cup C| \text{ is singles - doubles + triples} \]

\[ |A|+|B|+|C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| \]
Another counting rule: Pigeonhole Principle
Another counting rule:

**Pigeonhole Principle**

If there are $n$ pigeons with not enough holes for them to stay in ($k$ to be exact), what can we say about at least how many pigeons at least one hole will hold?
Another counting rule: **Pigeonhole Principle**

If there are $n$ pigeons with not enough holes for them to stay in (k to be exact), what can we say about at least how many pigeons at least one hole will hold?

$\text{ceil}(n / k)$
Another counting rule:

**STARS AND BARS**
Another counting rule: **STARS AND BARS**

How many ways can you distribute $n$ indistinguishable balls into $k$ distinguishable bins?
Another counting rule: **STARS AND BARS**

How many ways can you distribute \( n \) indistinguishable balls into \( k \) distinguishable bins?

*Arrange \( n \) balls and \( k - 1 \) dividers*
Another counting rule: **STARS AND BARS**

How many ways can you distribute \( n \) indistinguishable balls into \( k \) distinguishable bins?

Arrange \( n \) balls and \( k - 1 \) dividers

\[
\binom{n + k - 1}{n}
\]
PROBABILITY!
PROBABILITY!

- **Sample Space:** The set of all possible outcomes of an experiment, denoted \( \Omega \) or \( S \)
- **Event:** Some subset of the sample space, usually a capital letter such as \( E \subseteq \Omega \)
- **Union:** The union of two events \( E \) and \( F \) is denoted \( E \cup F \)
- **Intersection:** The intersection of two events \( E \) and \( F \) is denoted \( E \cap F \) or \( EF \)
- **Mutually Exclusive:** Events \( E \) and \( F \) are mutually exclusive iff \( E \cap F = \emptyset \)
- **Complement:** The complement of an event \( E \) is denoted \( E^C \) or \( \overline{E} \) or \( \neg E \), and is equal to \( \Omega \setminus E \)
- **DeMorgan’s Laws:** \((E \cup F)^C = E^C \cap F^C\) and \((E \cap F)^C = E^C \cup F^C\)
Each probability is between 0 and 1 inclusive

Probabilities add to 1

If events are *mutually exclusive*,

\[ P(A \cup B \cup C) = P(A) + P(B) + P(C) \]

because there are no intersections
PROBABILITY!

- **Axioms of Probability**
  - **Non-negativity:** For any event \( E \), \( \mathbb{P}(E) \geq 0 \)
  - **Normalization:** \( \mathbb{P}(\Omega) = 1 \)
  - **Additivity:** If \( E \) and \( F \) are mutually exclusive events, then
    \[
    \mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F)
    \]

- **Corollaries of these axioms**
  - **Complementation:** \( \mathbb{P}(E) + \mathbb{P}(E^C) = 1 \)
  - **Monotonicity:** If \( E \subseteq F \), \( \mathbb{P}(E) \leq \mathbb{P}(F) \)
  - **Inclusion-Exclusion:** \( \mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F) \)

- **Equally Likely Outcomes:** If every outcome in a finite sample space \( \Omega \) is equally likely, and \( E \) is an event, then \( \mathbb{P}(E) = \frac{|E|}{|\Omega|} \)
An event is a subset of the sample space

$E \subseteq \Omega$

If each outcome in the sample space is *equally likely*, the probability of an event is

$$P(E) = \frac{|E|}{|\Omega|}$$

If the union of a set of mutually exclusive events is equal to the sample sets, those events *partition* the sample space.