CSE 312

SECTION 5

ZOO OF RANDOM VARIABLES

– Welcome back, everyone! –

AGENDA

01 ANNOUNCEMENTS

02 VARIANCE

03 INDEPENDENT RV

04 ZOO OF RANDOM VARIABLES

01

ANNOUNCEMENTS

SCHEDULE REMINDERS

HW 3 GRADES WERE RELEASED

(regrade requests open and close after a week)

HW4 WAS RELEASED

Due next Wednesday

02

LOE REMINDER

LOE

When working with linearity of expectation, remember to

first define the RVs and the summation relationships
don't worry how the individual RVs are distributed

then apply linearity of expectation and find each value

02

VARIANCE

Variance is a another property of RVs (like expectation) that measures how much the values in the RV “vary”

VARIANCE - how "different" are values from the expectation "on average"

$$Var(X) = E[(X - E(X))^2] = \Sigma_x (P(X=x) * (x - E(X))^2)$$

expected value of the squared distance between each RV outcome and the expected value of RV

add up all the squared distances weighted by their probabilities

Properties

$$Var(a \cdot X + b) = a^2 \cdot Var(X)$$
$$Var(X) = E[X^2] - (E[X])^2$$

03

Independent RV

What does independence mean for random variables?

Random variables X and Y are **independent** if –

$$P(X=x, Y=y) = P(X=x) \cdot P(Y=y)$$

Knowing the value of X doesn't help "guess" what Y is

it's a useful property! if X and Y are independent random variables then —

$$E(X \cdot Y) = E[X] \cdot E[Y]$$

$$Var(X + Y) = Var[X] + Var[Y]$$

Random variables X and Y are **independent** if –

$$P(X=x,\ Y=y) = P(X=x) \cdot P(Y=y)$$

Knowing the value of X doesn't help "guess" what Y is

Additionally, there's **independent and identically distributed (aka, "i.i.d.")** random variables

Identically distributed means the random variables **have the same pmf –**

$$P(X=k) = P(Y=k)$$ for any value k

For example, rolling a die twice, where X is the first roll number and Y is the second roll number

04

ZOO OF RV'S

zoo of discrete random variables!

Zoo of Discrete Random Variables

Random variables allow us to represent different random experiments/situations

We've seen how tedious computing pmfs, expectations, and variances can be.

There are some *common situations* that call for a random variable that is too complex to analyze, so we derive pmfs, expectations, and variance for this *“zoo” of RVs*.

UNIFORM

Models situations where each outcome is equally likely

X ~ Uniform(a, b) if X is equally likely
to take on any value between a and b

$$p_X(k) = \frac{1}{b-a+1} \qquad \mathbb{E}[X] = \frac{a+b}{2} \qquad Var(X) = \frac{(b-a)(b-a+2)}{12}$$

A random variable X representing the outcome of rolling a fair 6 sided dice

X~Uniform(1, 6)

choosing a random value between 1 and 6 with each outcome equally likely

Bernoulli (Indicator)

Models situations where the RV can take on 0 or 1 (whether success or not)

X ~ Bernoulli(p) if X is 1 with probability of p

$$p_X(k) = \begin{cases} p, & k = 1 \\ 1 - p, & k = 0 \end{cases} \qquad \mathbb{E}[X] = p \qquad Var(X) = p(1-p)$$

X represents whether outcome of rolling a fair 6 sided dice is even (1) or not (0)

X~Bernoulli(3/6)

probability of 3/6 for "success"

Binomial

models situations when we count the # times an event occurs in n tries

X ~ Binomial(n, p) means X represents the number of times an event with probability p happens after n trials

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k} \qquad \mathbb{E}[X] = np \qquad Var(X) = np(1-p)$$

X represents the number of times the dice rolled to a 6 during 9 dice rolls

X~Binomial(9,⅙)

probability of success (rolling a 6) on a single dice roll is ⅙, and 9 trials (rolls)

Geometric

models situations when we count the # trials until some event occurs

X ~ Geometric(p) means X represents the number of trials before success (an event with probability p happens)

$$p_X(k) = (1-p)^{k-1}p, \qquad \mathbb{E}[X] = \frac{1}{p} \qquad Var(X) = \frac{1-p}{p^2}$$

X represents the number of times we roll a 6 sided die, before it rolls a 6

X~Geometric(⅙)

on a single dice roll, there's a probability of ⅙ for success (that it rolls a 6)

NEGATIVE BINOMIAL

(RELATED TO GEOMETRIC)

MODELS SITUATIONS WHERE WE COUNT # TRIALS TO GET SOME NUMBER OF SUCCESSES

X ~ NegBin(r,p) means X represents the number of trials to get r successes (probability of success on a single trial is p)

$$p_X(k) = \binom{k-1}{r-1} p^r (1-p)^{k-r} \qquad \mathbb{E}[X] = \frac{r}{p} \qquad Var(X) = \frac{r(1-p)}{p^2}$$

X represents number of dice rolls before we get 4 rolls with a 6

X~NegBin(4, 1/6)

because we want to have 4 trials (rolls) with success, and a success (rolling 6) has probability 1/6

Poisson

Models situations with *time* - how many successes in a unit of time

X ~ Poisson(λ) means X represents the number of success in a unit of time, where λ is average rate of successes per unit of time

$$p_X(k) = e^{-\lambda}\frac{\lambda^k}{k!},$$

$$\mathrm{E}[X] = \lambda$$

$$Var(X) = \lambda$$

X represents number of people born during a particular minute

X~Poisson(λ)

where λ represents the average birth rate per minute

HYPERGEOMETRIC

Models situations with *choosing* - how many "successes" do you get when choosing without replacement

X ~ HypGeo(N,K,n) means X represents the number of successes out of n draws from N items with K successes

Number of ways you can choose n items with k successes

Number of ways you can choose n items from N

$$p_X(k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}} \qquad \mathbb{E}[X] = n\frac{K}{N} \qquad Var(X) = n \cdot \frac{K(N-K)(N-n)}{N^2(2N-1)}$$

X represents number of Kit-Kats we will get when drawing 30 candies from a bowl of 100 candies that contain 10 Kit-Kats

X~HypGeo(100, 10, 30)

because we draw 30 from 100 items with 10 successes (Kit-Kats)

Discrete vs Continuous Random var.

two “types” of random vars

discrete

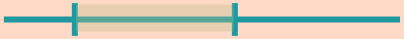
the range consists of finite/countably infinite values

discrete

the range consists of finite/countably infinite values

two “types” of random vars

continuous



the range consists of uncountably infinite values (*for example time is not discrete)*

discrete

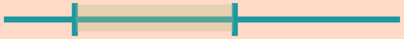
the range consists of finite/countably infinite values

PMF (prob. mass function)

$p_X(k) = P(X=k)$

two "types" of random vars

continuous



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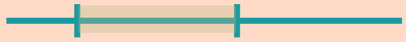


the range consists of uncountably infinite values (*for example time is not discrete)*

PMF (prob. mass function)

$p_X(k) = P(X=k) = 0$

two "types" of random vars

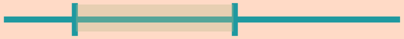
discrete

the range consists of finite/countably infinite values

PMF (prob. mass function)

$p_X(k) = P(X=k)$

continuous



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PDF (prob. ***density*** *function)*

$f_X(k) \mathrel{!=} P(X=k)$

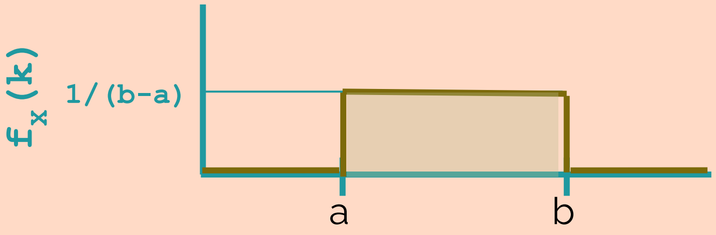
discrete vs. continuous

	Discrete	Continuous
PMF/PDF	$p_X(x) = P(X = x)$	$f_X(x) \neq P(X = x) = 0$
CDF	$F_X(x) = \sum_{t \leq x} p_X(t)$	$F_X(x) = \int_{-\infty}^{x} f_X(t)\, dt$
Normalization	$\sum_{x} p_X(x) = 1$	$\int_{-\infty}^{\infty} f_X(x)\, dx = 1$
Expectation	$\mathbb{E}[g(X)] = \sum_{x} g(x)\, p_X(x)$	$\mathbb{E}[g(X)] = \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx$

Zoo of continuous RVs

Uniform RV (continuous version)

X~Unif(a, b) randomly takes on any real number between a and b



$$f_X(x) = \begin{cases} \frac{1}{b-a} & \text{if } x \in [a, b] \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbb{E}[X] = \frac{a+b}{2}$$

$$\mathrm{Var}(X) = \frac{(b-a)^2}{12}$$

Exponential RV

X~Exp(λ) tells how much time till a certain event happens
(λ *is the rate of time*)

think of this as the "continuous version" of the geometric distribution!

don't confuse this with the Poisson distribution just bc it's related with time, they're very different!
(Poisson is *number* of events in a certain time frame)

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbb{E}[X] = \frac{1}{\lambda}$$

$$\text{Var}(X) = \frac{1}{\lambda^2}$$

$$F_X(x) = 1 - e^{-\lambda x}$$

$F_X(x)$ = P(X <= x) *this is the integral of* $f_X(x)$