more MLE

LECTURE 21

1. DEFINE LIKELIHOOD FUNCTION

$\mathcal{L}(E;\theta)$ is $\mathbb{P}(E)$ when the experiment is run with θ

Discrete Distribution

$\mathcal{L}(x_1, x_2, \ldots, x_n; \theta) = \Pi_i^n \mathbb{P}(x_i; \theta)$

Continuous Distribution

$\mathcal{L}(x_1, x_2, \ldots, x_n; \theta) = \Pi_i^n f_X(x_i; \theta)$

Multiple Parameters

$\mathcal{L}(x_1, \ldots, x_n; \theta_1, \theta_2)$

now the likelihood will be in terms of multiple unknown

2. TAKE LOG OF THE LIKELIHOOD (LOG-LIKELIHOOD

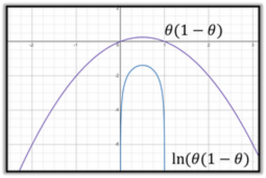
The product rule is not fun!! So, take the log of the likelihood function before taking the derivative!

*ln(a*b)=ln(a)+ln(b)*

Can we still take the max? Yes! ln() is an increasing function, so

$\text{argmax}_\theta \ln(\mathcal{L}(E;\theta)) = \text{argmax}_\theta \mathcal{L}(E;\theta)$

"the log of the likelihood will increase as the likelihood increases and vice verse, so, the value of θ that maximizes the log likelihood also maximized the likelihood"

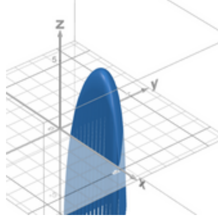


Coin Example: $\ln(\mathcal{L}(\text{HTTTHHTHHH}; \theta) =$

Now, we want to find the value of theta, that will maximize the likelihood (maximize the probability of the data)

The MLE of the parameter θ is: $\hat{\theta} = \text{argmax}_\theta \ \mathcal{L}(E;\theta)$

If solving for MLE with 2 parameters, look for the *pair* of values that give the max likelihood



3. TAKE DERIVATIVE(S) OF THE LOG-LIKELIHOOD

If solving for MLE with 2 parameters, take **partial derivatives** with respect to each parameter

4. SET DERIVATIVE(S) TO 0, SOLVE FOR THE MLE

If solving for MLE with 2 parameters, set *both* derivatives to 0, and solve system of equations.

5. 2ND DERIVATIVE TEST TO VERIFY MAX

Suppose you get values $x_1, x_2, \ldots x_n$ from independent draws of a normal random variable $\mathcal{N}(\mu, 1)$ (for an unknown μ). Find the MLE for μ.

1. Write the likelihood function:

2. Take the log of the likelihood function

3. Take the derivatives of the log-likelihood function

4. Set the derivatives to 0, and solve for the MLE $\theta\hat{}$

5. Verify it is a maximum with second derivative test (not required for 312)

Let θ_μ and θ_{σ^2} be the unknown mean and variance of a normal distribution. We get independent draws $x_1, x_2, \ldots, x_n$ from the distribution. Find the MLEs for $\theta_\mu$and θ_{σ^2}.

1. Write the likelihood function:

2. Take the log of the likelihood function

3. Take the derivatives of the log-likelihood function

4. Set the derivatives to 0, and solve for the MLE $\theta\hat{}$

5. Verify it is a maximum with second derivative test (not required for 312)