You run a poll of 1000 people where 60% of true population supports you. What is the probability that the poll is not within 10% of the true value? 

1. bound the left tail 
2. bound the right tail 
3. putting it together 

There are 20 frogs on each location in a 5x5 grid. Each frog independently jumps L, R, U, D, or neither with equal probability. Bound the probability at least one square ends up with at least 36 frogs.

1. Apply Union Bound 
2. Apply Chernoff Bound to bound each of P(Ai) 
3. Put it all together 

**MAXIMUM LIKELIHOOD ESTIMATION**

**Goal:** derive an estimate \( \hat{\theta} \) for the parameter \( \theta \) based on observed data

1. Run the experiment a bunch of times (i.e., collect many samples from the distribution) \( \rightarrow \) data 
   e.g., we flip a coin that follows Ber(\( p \)) 5 times and write down the results - HTTTH 

2. Estimate the missing rules (unknown parameter(s)) based on the data 
   \( \rightarrow \) Guess rules to maximize probability of the events we saw (relative to other choices of the rules). 
   e.g., what is the value of \( p \) that makes the probability of seeing HTTTH the highest?
We're given that there's a distribution with some unknown parameter(s) \( \theta \). There are independent observations \( x_1, x_2, \ldots, x_n \) from this distribution. To find the MLE for this unknown parameter(s) \( \theta \), follow these steps:

1. **Write the likelihood function**
   - Multiply (not add) probabilities of seeing each of the observations based on \( \theta \).

2. **Take the log** of the likelihood function (makes the math easier)

3. **Take the derivative** of the log-likelihood function

4. **Set the derivative to 0**, and solve for the MLE \( \hat{\theta} \)
   - Remember to switch from \( \theta \) to \( \hat{\theta} \) in this step because we’re now solving for the MLE.

5. **Verify it is a maximum with second derivative test** (not required for 312)

**Coin Example:**
We ran the experiment 10 times independently. The result was HTTHHTHHTH.
\[
\mathcal{L}(\text{HTTHHTHHTH}; \theta) = \text{“Probability of observing HTTHHTHHTH if \theta is probability of heads on a single flip”}
\]

**General Steps**
We’re given that there’s a distribution with some unknown parameter(s) \( \theta \). There are independent observations \( x_1, x_2, \ldots, x_n \) from this distribution.

To find the MLE for this unknown parameter(s) \( \theta \),...

1. Write the likelihood function
   - Multiply (not add) probabilities of seeing each of the observations based on \( \theta \)

2. Take the log \( \ln(\cdot) \) of the likelihood function (makes the math easier)

3. Take the derivative of the log-likelihood function

4. Set the derivative to 0, and solve for the MLE \( \hat{\theta} \)
   - Remember to switch from \( \theta \) to \( \hat{\theta} \) in this step because we’re now solving for the MLE.

5. Verify it is a maximum with second derivative test (not required for 312)