more CLT + joint distributions

LECTURE 16

Central Limit Theorem can be used if we are finding values that will give a certain probability. We do the same process and end up with a reverse table lookup

Joint distributions allow us to analyze the relationships between multiple random variables

	Discrete	Continuous
Joint PMF/PDF	$p_{X,Y}(x,y) = P(X=x, Y=y)$	$f_{X,Y}(x,y) \neq P(X=x, Y=y)$
Joint CDF	$F_{X,Y}(x,y) = \sum_{t \le x} \sum_{s \le y} p_{X,Y}(t,s)$	$F_{X,Y}(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{X,Y}(t,s) ds dt$
Normalization	$\sum_x \sum_y p_{X,Y}(x,y) = 1$	$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y) dx dy = 1$
Marginal PMF/PDF	$p_X(x) = \sum_y p_{X,Y}(x,y)$	$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dy$
Expectation	$E[g(X,Y)] = \sum_x \sum_y g(x,y) p_{X,Y}(x,y)$	$E[g(X,Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f_{X,Y}(x,y) dx dy$
Independence	$\forall x, y, p_{X,Y}(x,y) = p_X(x) p_Y(y)$	$\forall x, y, f_{X,Y}(x,y) = f_X(x) f_Y(y)$

CLT EXPAMPLE (DOING IT IN REVERSE)

A cereal company claims that their boxes contain an average of 500 grams of cereal with a variance of 100 grams. To test this claim, you take a random sample of n boxes. You want to determine the sample size n such that the probability of the sample mean being within 2 grams of the true mean (i.e., between 498 grams and 502 grams) is at least 95%.

1. Setup the Problem.

2. Apply CLT

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3. Compute Probability

Reverse z-table Lookup

In problems where we need to find the value of something that will make certain probability statement hold, we may follow the same CLT steps, but then do a reverse z-table lookup

Phi(c)=0.975 ---> *"what value do we plug into the z table to get 0.95?"* --> Phi^-1(0.975)=1.96

JOINT DISTRIBUTIONS

JOINT SUPPORT

Set of pairs of values X and Y can be at the same time

$$\Omega_{X,Y} = \{(a,b) : p_{X,Y}(a,b) > 0\} \subseteq \Omega_X \times \Omega_Y$$

JOINT PMF

Probabilities of X and Y being certain values

$$p_{X,Y}(a,b) = \mathbb{P}(X = a \cap Y = b) = \mathbb{P}(X = a, Y = b)$$

should be defined for ***all*** *values of* a *and* b

JOINT CDF

$$F_{X,Y}(a,b) = \mathbb{P}(X \le a \cap Y \le b) = \mathbb{P}(X \le a, Y \le b)$$

should be defined for ***all*** *values of* a *and* b

JOINT INDEPENDENCE

X and Y are independent if:

> $p_{X,Y}(a,b) = p_X(a) \cdot p_X(b)$ for all $(a,b) \in \Omega_{X,Y}$

> $\Omega_{X,} = \Omega_X \times \Omega_Y$

MARGINAL PMF

$$p_X(a) = \sum_{b \in \Omega_Y} p_{X,Y}(a,b)$$

JOINT EXPECTATION

$$\mathbb{E}[g(X,Y)] = \sum_{(a,b) \in \Omega_{X,Y}} g(a,b) \cdot p_{X,Y}(a,b)$$

Random experiment: Roll two 4-sided fair. ***X*** *and* ***Y*** *are the values of each of the rolls.*

Joint Support of X and Y:

$\Omega_{X,Y} =$

Joint PMF of X and Y:

$p_{X,Y}(a,b) = \{$

$p_{X,Y}$	Y=1	Y=2	Y=3	Y=4
X=1	1/16	1/16	1/16	1/16
X=2	1/16	1/16	1/16	1/16
X=3	1/16	1/16	1/16	1/16
X=4	1/16	1/16	1/16	1/16

U *is the min value of the two dice min(X,Y)*

V *is the max vlaue of the two dice max(X,Y)*

Joint support of U and V:

$\Omega_{U,V} =$

Joint PMF of U and V:

$p_{U,W}(u,w) = \{$

$p_{U,V}$	V=1	V=2	V=3	V=4
U=1				
U=2				
U=3				
U=4				

U and W are NOT independent because $\Omega_{U,W} \neq \Omega_U \times \Omega_W$

Suppose we didn't know how to compute the PMF for U. Can we derive it from the joint PMF?

Use LoTP, partitioning on V=1, V=2, V=3, V=4

$$\mathbb{P}(U = 1)$$
$$= \mathbb{P}(U = 1 \cap V = 1) + \mathbb{P}(U = 1 \cap V = 2)$$
$$+ \mathbb{P}(U = 1 \cap V = 3) + \mathbb{P}(U = 1 \cap V = 4)$$

(CONTINUOUS) JOINT DISTRIBUTIONS

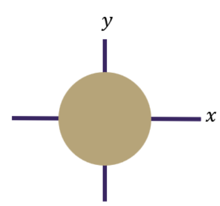
	Continuous
Joint PMF/PDF	$f_{X,Y}(x,y) \neq P(X = x, Y = y)$
Joint CDF	$F_{X,Y}(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{X,Y}(t,s)dsdt$
Normalization	$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y)dxdy = 1$
Marginal PMF/PDF	$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dy$
Expectation	$E[g(X,Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f_{X,Y}(x,y)dxdy$

Replace sums with integrals, PMF with PDF

X and Y are coordinates of the where the dart lands at.

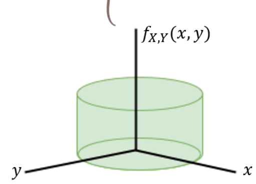
Joint Support

$\Omega_{X,Y} =$



Joint PDF

$f_{X,Y}(x,y) = \{$



Marginal PDF

$f_X(x) =$