zoo of continuous RVs

LECTURE 14

ZOO OF CONTINUOUS RANDOM VARIABLES

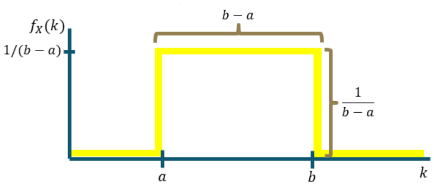
> **Uniform *Continuous* Distribution**: Unif(a,b) ~ real number selected uniformly at random between a and b
> **Exponential Distribution** : Exp(lambda) ~ time till first success, where lambda is rate of success
> **Normal Distribution**: N(mu, sigma^2) is a normal distribution with mean mu and variance sigma^2
- we will see this in a lot of real world situtations (spoiler: CLT!) - it looks like a bell curve
- to compute probabilites (1) *standardize* (2) *write in terms of Phi(z)* (3) *plug into the z-table*

CONTINUOUS UNIFORM DISTRIBUTION

X is a <u>uniform random real number</u> between a and b -> X~Unif(a,b)

Probability Density Function (PDF)

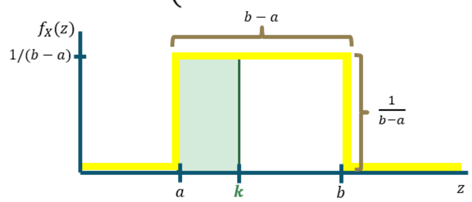
PDF: $f_X(k) = \begin{cases} \frac{1}{b-a} & \text{if } a \le k \le b \\ 0 & \text{otherwise} \end{cases}$



.Area under this curve is 1

Cumulative Distribution Function

CDF: $F_X(k) = \begin{cases} 0 & \text{if } k < a \\ \frac{k-a}{b-a} & \text{if } a \le k \le b \\ 1 & \text{if } k \ge b \end{cases}$



Expectation

Expectation: $\mathbb{E}[X] = \frac{a+b}{2}$

$\mathbb{E}[X] = \int_{-\infty}^{\infty} z \cdot f_X(z)\,\mathrm{dz}$ 1/(b

$= \int_{-\infty}^{a} z \cdot 0\,\mathrm{dz} + \int_{a}^{b} z \cdot \frac{1}{b-a}\mathrm{dz} + \int_{b}^{\infty} z \cdot 0\,\mathrm{dz}$

Variance

Variance: $\mathrm{Var}(X) = \frac{(b-a)^2}{12}$

$\mathrm{Var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$

$=$

EXPONENTIAL DISTRIBUTION

X~Exp(λ) is <u>time till the first event</u>. Average of λ events per time unit.

This sounds very similar to a <u>geometric</u> distribution!
> Geometric random variable is the **number of trials** till success (discrete).
> Exponential random variable is *time* (a real number, continuous) till success

Similar to geometric, exponential RVs **<u>memoryless</u>**: **P(X>=k+1|X≥1)=P(Y≥k)**

Cumulative Distribution Function

$F_X(k) = \begin{cases} 1 - e^{-\lambda k} & \text{if } k \ge 0 \\ 0 & \text{otherwise} \end{cases}$

$F_X(t) = 1 - \mathbb{P}(X > t)$ "its take more than t time units for first event"

$= 1 - \mathbb{P}(Y = 0)$ = "0 successes in the first t time units"

$= 1 - e^{-\lambda t}\frac{(\lambda t)^0}{0!}$

$= 1 - e^{-\lambda t}$

Probability Density Function - take the *derivative* of the CDF

PDF: $f_X(k) = \begin{cases} \lambda e^{-\lambda k} & \text{if } k \geq 0 \\ 0 & \text{otherwise} \end{cases}$

Expectation

Expectation: $\mathbb{E}[X] = \frac{1}{\lambda}$

Variance

Variance: $\text{Var}(X) = \frac{1}{\lambda^2}$

NORMAL DISTRIBUTION

Normal RVs appear a lot in the real world! It's defined below:

A **normal random variable** $X \sim \mathcal{N}(\mu, \sigma^2)$ has two parameters:

- $\mu = \mathbb{E}[X]$ is the mean
- $\sigma^2 = \text{Var}(X)$ is the variance ($\sigma = \sqrt{Var(X)}$ is *standard deviation*)

and follows this ***probability density function*** (a bell curve!):

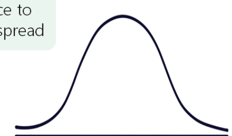
$$f_X(k) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(k-\mu)^2}{2\sigma^2}}$$

constant for normalization

symmetric around the mean

variance to control spread

exponential term for tails



Properties of Normal RVs

When we ***scale or shift a normal***, we get a normal random variable

If $X \sim \mathcal{N}(\mu, \sigma^2)$

Then for $Y = aX + b$, $Y \sim \mathcal{N}(a\mu + b, a^2\sigma^2)$

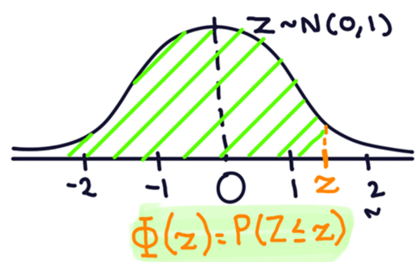
When we ***add independent normals***, we get a normal random variable

If $X \sim \mathcal{N}(\mu_X, \sigma_X^2)$ and $Y \sim \mathcal{N}(\mu_Y, \sigma_Y^2)$ and X and Y are independent,

Then, for $Z = aX + bY + c$, $Z \sim \mathcal{N}(a\mu_X + b\mu_Y + c, a^2\sigma_X^2 + b^2\sigma_Y^2)$

HOW TO FIND PROBABILITIES (E.G., CDF) OF A NORMAL RANDOM VARIABLE?

The **z-table** contains values for the CDF of the standard normal random variable Z~N(0,1)



1. Write the probability we're interested in in terms of the CDF
2. Standardize the normal random variable: Z=(X-μ)/σ
3. Round the "z-score"(s) to the hundredths place.
4. Look up the value(s) in the table