discrete zoo II

LECTURE 11

Uniform: Every integer between a and b are equally likely
Bernoulli: Whether there is success in one trial
Binomial: Number of successes in n independent trials
Geometric: Number of trials till first success
Poisson: Number of successes in a time interval
Negative Binomial: Number of trials till r'th success
Hypergeometric: Number of successes when drawing a sample

ZOO OF DISCRETE RANDOM VARIABLES II

GEOMETRIC DISTRIBUTION

The number of independent trials until the first success

$X \sim \text{Geo}(p)$

p is the probability of success for one trial.

$\Omega_X = \{1,2,3,4,\ldots\}$

PMF: $p_X(k) = (1-p)^{k-1}p$ for $k \in \{1,2,3,\ldots\}$

CDF: $F_X(k) = 1-(1-p)^k$ for $k \in \mathbb{N}$

Expectation: $\mathbb{E}[X] = \frac{1}{\text{p}}$

Variance: $\text{Var}(X) = \frac{1-p}{p^2}$

MEMORYLESS

P(X >= a+b | X >= a) = P(X >= b)

POISSON DISTRIBUTION

The number of success/incidents in a certain time interval

$X \sim \text{Poi}(\lambda)$

PMF: $p_X(k) = \frac{\lambda^k e^{-\lambda}}{k!}$ (for $k \in \mathbb{N}$)

CDF: $F_X(k) = e^{-\lambda} \sum_{i=0}^{\lfloor k \rfloor} \frac{\lambda^i}{i!}$

Expectation: $\mathbb{E}[X] = \lambda$

Variance: $\text{Var}(X) = \lambda$

examples:

NEGATIVE BINOMIAL DISTRIBUTION

Number of independent trials till the r'th success

$X \sim \text{NegBin}(r, p)$

PMF: $p_X(k) = \binom{k-1}{r-1}(1-p)^{k-r}p^r$

CDF: $F_X(k)$ is ugly, don't bother with it

Expectation: $\mathbb{E}[X] = \frac{r}{p}$

Variance: $\text{Var}(X) = \frac{r(1-p)}{p^2}$

examples:

Deriving the PMF

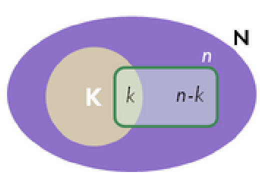
Deriving the Expectation

Deriving the Variance

HYPERGEOMETRIC DISTRIBUTION

You have an urn with N balls, of which K are targets. You are going to draw n balls out of the urn without replacement. How many target balls do we get in this sample?

Deriving the PMF



Deriving the Expectation