variance + discrete zoo

# LECTURE 10

**> INDEPENDENCE OF RANDOM VARIABLES:**

$X$ and $Y$ are independent if for all $k, \ell$

$\mathbb{P}(X = k, Y = \ell) = \mathbb{P}(X = k)\mathbb{P}(Y = \ell)$

**> EXPECTATION OF A FUNCTION OF X:**

$$\mathbb{E}[g(X)] = \sum_{k \in \Omega_X} g(k) \cdot \mathbb{P}(X = k)$$

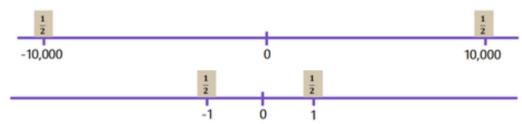
**> LINEARITY OF EXPECTATION:**

***E[aX+bY+c]=aE[X]+bE[Y]+c***

## TODOS FOR WEEK 4

- ☐ Concept check 8 ☐ 9 ☐ 10
- ☐ HW3 due on Wed (7/11) ☐ Coding on Fri. (7/13)
- ☐ Read midterm information
- ☐ Practice for midterm, review session on Friday
- ☐ Attend/participate in section on Thursday

***Coming up with the variance measure...***




## VARIANCE

***Variance is a quantity that measures on average how "far" the random variable is from its expectation***

$$\mathrm{Var}(X) = \sum_{\omega} \mathbb{P}(\omega) \cdot (X(\omega) - \mathbb{E}[X])^2 = \mathbb{E}[(X - \mathbb{E}[X])^2]$$

$$= \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

**Let X be the result of rolling a fair die. What is Var(X)?**

## VARIANCE ADDS FOR INDEPENDENT RANDOM VARIABLES

**Var(X+Y)=Var(X)+Var(Y) IF X and Y are independent**

**Let X be the number of heads in n coin flips (probability p of heads). What is Var(X)?**

## MORE PROPERTIES OF INDEPENDENCE

**Var(X+c)=Var(X)** *shifting*

**Var(aX)=a²Var(X)** *scaling*

# ZOO OF DISCRETE RANDOM VARIABLES

THE DISCRETE ZOO DEFINES SOME COMMON PATTERNS AND GIVES US THE PMF, EXPECTATION, AND VARIANCE, SO WE DON'T HAVE TO COMPUTE IT EVERY TIME!

## UNIFORM DISTRIBUTION

***You want an integer in some range, with each integer equally likely.***

$X \sim \text{Unif}(a, b)$

PMF: $p_X(k) = \frac{1}{b-a+1}$ for $k \in \mathbb{Z}, a \leq k \leq b$

CDF: $F_X(k) = \frac{k-a+1}{b-a+1}$ for $k \in \mathbb{Z}, a \leq k \leq b$.

Expectation: $\mathbb{E}[X] = \frac{a+b}{2}$

Variance: $\text{Var}(X) = \frac{(b-a)(b-a+2)}{12}$

## BERNOULLI DISTRIBUTION

***Do I get a success in one trial with probability p of "success"?***

$X \sim \text{Ber}(p)$

$X$ is the indicator random variable that the trial was a success.

Parameter $p$ is probability of success on the trial.

PMF: $p_X(0) = 1 - p$, $p_X(1) = p$

CDF: $F_X(k) = \begin{cases} 0 & \text{if } k < 0 \\ 1-p & \text{if } 0 \leq k < 1 \\ 1 & \text{if } k \geq 1 \end{cases}$

Expectation: $\mathbb{E}[X] = p$

Variance: $\text{Var}(X) = p(1-p)$

## BINOMIAL DISTRIBUTION

***How many success did you see in n independent trials, where each trial has probability p of success?***

$X \sim \text{Bin}(n, p)$

$X$ is the number of successes across $n$ independent trials.

$n$ is the number of independent trials.
$p$ is the probability of success for one trial.

PMF: $p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$ for $k \in \{0, 1, \ldots, n\}$

CDF: $F_X$ is ugly.

Expectation: $\mathbb{E}[X] = np$

Variance: $\text{Var}(X) = np(1-p)$

*example:*