

"knowing the value of one RV doesn't tell us anything about what the value of the other might be X and Y are independent if for all k, ℓ $\mathbb{P}(X = k, Y = \ell) = \mathbb{P}(X = k)\mathbb{P}(Y = \ell)$

Are S="the sum of two dice" and R="the value of the red die" independent?

X ~ "the # of heads in the first n flips." Y ~ "the # of heads in the last n flips." Are X and Y independent?

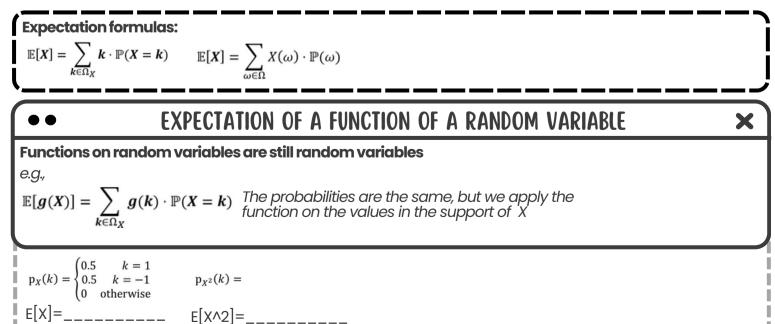
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MUTUAL INDEPENDENCE OF RANDOM VARIABLES

X

 X_1, X_2, \dots, X_n are mutually independent if for all x_1, x_2, \dots, x_n $\mathbb{P}(X_1 = x_1, X_2 = x_2, \dots, X_n = x_n) = \mathbb{P}(X_1 = x_1)\mathbb{P}(X_2 = x_2) \cdots \mathbb{P}(X_n = x_n)$

MORE OF EXPECTATION!



••	LINEARITY OF EXPECTATION	
The expected value of a sum of random variables is the sum of their individual expectations ★E[X+Y]=E[X]+E[Y] ★E[X1+X2++Xn]=E[X1]+E[X2]++E[Xn] ★E[aX+bY+c]=aE[X]+bE[Y]+c		
How many fish do bot > E[] =	$\mathbb{E}[X]$ =3. Your friend catches <i>Y</i> fish, with $\mathbb{E}[Y]$ th of you bring on an average day? \$10, but you need \$15 for expenses. What	
••		 X

<i>v</i> –	ſ	1
<i>л</i> –	l	0

if event A occurs otherwise $p_X(k) = \begin{cases} \mathbb{P}(A) & \text{if } k = 1\\ 1 - \mathbb{P}(A) & \text{if } k = 0\\ 0 & \text{otherwise} \end{cases}$

 $\mathbb{E} [X] = 1 \cdot p_X(1) + 0 \cdot p_X(0)$ = $p_X(1) = \mathbb{P}(A)$

GENERAL PROCESS FOR USING LOE WHEN WE HAVE MORE COMPLEX SITUATIONS

> Decompose: Decompose the random variable into sum of simple RVs: X=X1+X2+...+Xn

> LOE: Apply Linearity of Expectation \square – E[X]=E[X1]+E[X2]+...+E[Xn]

> Conquer: Compute the expectation of each Xi

EXAMPLES...

The probability of flipping a head is p and we flip a coin n times. X ~ total number of heads seen What is E[X]?

In a class of m students, on average how many pairs of people have the same birthday? **Decompose:** Let X be the number of pairs with the same birthday

LOE:

<u>Conquer</u>:

n people are sitting around a circular table. There is a name tag in each place. Nobody is sitting in front of their own name tag. Rotate the table by a random number k of positions between 1 and n-1 (equally likely). Let X be the number of people that end up in front of their own name tag. Find E[X]. **Decompose:**

LOE:

<u>Conquer</u>: