random variables

# LECTURE 8

***RANDOM VARIABLES: assign quantitative values to outcomes of a random experiment.***
***› RANGE/SUPPORT: $\Omega_X$ is set of possible values X can be***
***› PROBABILITY MASS FUNCTION: $p_X(k)=P(X=k)$ › CUMULATIVE DISTRIBUTION FUNCTION: $F_X(k)=P(X\leq k)$***
***› EXPECTATION: Average of values on the support, weighted on the probabilities X takes on each***
***› VARIANCE: Coming soon!***

## RANDOM VARIABLES

### RANDOM VARIABLES

***Summarizes Important information from outcomes from a sample space.***

$\Omega$ is set of possible outcomes

**Outcome** $\omega \in \Omega$ → $X$ → **Quantitative value**

e.g.,

***Notation notes:***
› Use a capital letter to denote a random variables
› X=k represents the set of outcomes that the random variable X assigns the value k

## DESCRIBING RANDOM VARIABLES

### SUPPORT - WHAT VALUES CAN THE RANDOM VARIABLES TAKE?

***Support/range is the set of values X could possibly take***

*Random experiment: Roll a fair red and fair blue dice*
**D** (difference of red and blue dice) has support _____
**S** (sum of the two dice) has support _____
**M** (max of the two dice) has support _____

### PROBABILITY MASS FUNCTION (PMF)

***The probability of X being each of the possible values - $p_X(k) = P(X=k)$***

T ~ number of 2's when we roll a red and blue dice

PMF for T: **$P(X=k) = p_X(k) =$**

**A random variable partitions the sample space**
X=0, X=1, X=2, etc. are all *events* that partition the sample space

$$\sum_{k \in \Omega_x} p_X(k) = 1$$

## CUMULATIVE DISTRIBUTION FUNCTION (CDF)

***The probability of X being less than or equal to a value k - F_X(k) = P(X<=k)***

## EXAMPLE

There are 20 balls, numbered 1, 2, ..., 20. You'll draw out a size-3 subsets (uniformly at random).
*X is a random variable that is the largest value among the three balls*

> What is the **support** of X?
> What is the **PMF** of X?

> What is the **CDF** of X?

## EXPECTATION

***The weighted average over all the values in the support X, weighted on the probability of each***

$$\mathbb{E}[X] = \sum_{k \in \Omega_X} k \cdot \mathbb{P}(X = k)$$

**Flip a fair coin twice (independently). What is the expected number of heads we see?**
*1. Find the PMF for X* *2. Compute E[X]*

**A die shows a 6 with probability 1/3, and 1,...,5 with probability 2/15 each. X is value of the die. What is E[X]?**
*1. Find the PMF for X* *2. Compute E[X]*

**Let X be the result shown on a fair die. What is E[X]?**
*1. Find the PMF for X* *2. Compute E[X]*

**Let Y be the sum of two (independent) fair die rolls. What is E[Y]?**
*1. Find the PMF for X* *2. Compute E[X]*