conditional probability

# LECTURE 5

***P(A|B) is the probability that the event A occurs conditioning on/given that B occurred***

***DEFINITION OF CONDITIONAL PROBABILITY:*** $\mathbb{P}(\boldsymbol{A}|\boldsymbol{B}) = \frac{\mathbb{P}(\boldsymbol{A} \cap \boldsymbol{B})}{\mathbb{P}(\boldsymbol{B})}$

***BAYES' RULE:*** $\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$

***LAW OF TOTAL PROBABILITY (LOTP):*** $\mathbb{P}(A) = \sum_{\text{all } i} \mathbb{P}(A \cap E_i) = \sum_{\text{all } i} \mathbb{P}(E_i|A_i)\mathbb{P}(A_i)$

**There are about 50 people in this class. What is the probability that at least 2 of us share a birthday? Assume there are exactly 365 possible birthdays, and each possibility is equally likely.**

*Sample Space:*

*Probability Measure:*

*Event:*

*Probability:*

***Takeaways/Reminders:***

• If you're dealing with a situation where you may be able to use a uniform probability space, make sure to set up the sample space in a way that every outcome is equally likely.•

•Try not overcomplicate the sample space – only include the information that you need in it. •

•When you define an event, make sure it is a subset of the sample space!

## CONDITIONAL PROBABILITY

FINDING PROBABILITIES WHEN WE'RE GIVEN SOME PARTIAL INFORMATION ABOUT THE OUTCOME

### DEFINITION OF CONDITIONAL PROBABILITY

***For an event B, with P(B)>0, the "Probability of A conditioned on B" is:*** $\mathbb{P}(\boldsymbol{A}|\boldsymbol{B}) = \frac{\mathbb{P}(\boldsymbol{A} \cap \boldsymbol{B})}{\mathbb{P}(\boldsymbol{B})}$

| | D2=1 | D2=2 | D2=3 | D2=4 | D2=5 | D2=6 |
|---|---|---|---|---|---|---|
| D1=1 | (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1.6) |
| D1=2 | (2,1) | (2,2) | (2,3) | (2,4) | (2,5) | (2,6) |
| D1=3 | (3,1) | (3,2) | (3,3) | (3,4) | (3,5) | (3,6) |
| D1=4 | (4,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4,6) |
| D1=5 | (5,1) | (5,2) | (5,3) | (5,4) | (5,5) | (5,6) |
| D1=6 | (6,1) | (6,2) | (6,3) | (6,4) | (6,5) | (6,6) |

A ~ Red die 6
B ~ Sum is 7
C ~ Sum is 9

P(A | B) =
P(A | C) =
P(B | A) =

DOES DIRECTION MATTER? IS P(A|B)=P(B|A)?

# WONKA BARS

**Willy Wonka has placed golden tickets on 0.1% of his Wonka Bars.**

*You want to get a golden ticket. You could buy a 1000-or-so of the bars until you find one, but that's expensive...you've got a better idea!*

*You have a test – a very precise scale you've bought.*

**If the bar you weigh does have a golden ticket, the scale will alert you 99.9% of the time.**

**If the bar you weigh does not have a golden ticket, the scale will alert you 1% of the time.**

**If you pick up a bar and it alerts, what is the probability you have a golden ticket?**

**P(  ) = 0.001**

**P(  ) = 0.999**

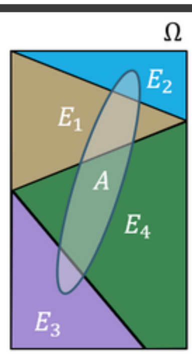
**P(  ) = 0.01**

**P(  ) = ?**

## BAYES' RULE

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$$

## LAW OF TOTAL PROBABILITY




**Events E1, E2, ..., En partition a sample space if**
**(1) they don't overlap and (2) their union is the sample space**

$$\mathbb{P}(A) = \sum_{\text{all } i} \mathbb{P}(A \cap E_i) = \sum_{\text{all } i} \mathbb{P}(E_i|A_i)\mathbb{P}(A_i)$$

So.... P(A) =

# OTHER TAKEAWAYS...

**After you condition on an event, what remains is a probability space.**

(Ω,P) is a valid probability space

after conditioning on A, (A,P(·|A)) is valid probability space

$$\sum_{\omega \in A} \mathbb{P}(\omega|A) = 1$$

$$\mathbb{P}(\bar{E}|A) = 1 - \mathbb{P}(E|A)$$