probability

LECTURE 4

RANDOM EXPERIMENT: An experiment where the outcome can't be predicted with certainty beforehand.
SAMPLE SPACE: Ω is set of possible outcomes ***EVENT:*** $E \subseteq \Omega$ is a subset of possible outcomes
PROBABILITY SPACE: A pair (Ω, P) where Ω is the sample space and P is the probability measure
UNIFORM PROBABILITY SPACE: A (common) probability space where very outcome in the sample space is equally likely to occur. In uniform probability spaces:
> For every *outcome* $\omega \in \Omega$, $\mathbb{P}(\omega) = \frac{1}{|\Omega|}$ > For an *event* $E \subseteq \Omega$, $\mathbb{P}(E) = \frac{|E|}{|\Omega|}$

PROBABILITY

A WAY OF QUANTIFYING OUR UNCERTAINTY WHEN MORE THAN ONE OUTCOME IS POSSIBLE

EXPERIMENT

An action or process that leads to one or more outcomes. A ***random experiment*** is an experiment where the outcome can't be predicted with certainty beforehand.

e.g., ___________________________

SAMPLE SPACE

Ω **is set of possible outcomes in an experiment**

e.g., ___________________________

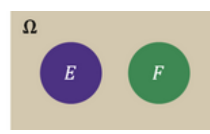
Example...

EVENT

An event $E \subseteq \Omega$ **is a <u>subset</u> of possible outcomes**

e.g., ___________________________

Two events E,F are ***<u>mutually exclusive</u>*** if they can't happen simultaneously.



PROBABILITY

Probability is a number between 0 and 1 describing how likely a particular outcome or event is.

ω (a particular outcome - an element of Ω) → $\mathbb{P}$ → $\mathbb{P}(\omega)$ - A value in [0,1] (the probability of that outcome)

Function that assigns probabilities to each outcome
e.g.,

PROBABILITY SPACE

A (discrete) probability space is a pair (Ω, P) where Ω is the <u>sample space</u> and P is the <u>probability measure</u>

Requirements of a *valid probability measure*:
1. All probabilities are between 0 and 1
2. The sum the probabilities of each outcome is 1

e.g.,

FINDING THE PROBABILITY OF AN EVENT

The probability measure gives the probability of each individual outcome. To get the probability of an *event* (set of outcomes), we sum together the probabilities of each individual outcome:

$$\boxed{\mathbb{P}(E) = \sum_{\omega \in E} \mathbb{P}(\omega)}$$

e.g.,

PROBABILITY FACTS (THAT FOLLOW FROM PROBABILITY MEASURE REQUIREMENTS)

- $\mathbb{P}(\bar{E}) = 1 - \mathbb{P}(E)$ (complementation)
- If $E \subseteq F$, then $\mathbb{P}(E) \leq \mathbb{P}(F)$ (monotonicity)
- $\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F)$ (inclusion-exclusion)
  - if E and F are mutually exclusive: $\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F)$

UNIFORM PROBABILITY SPACES

UNIFORM PROBABILITY SPACE

A probability space where ***all outcomes in the sample space are equally likely to occur***.

> For every *outcome* $\omega \in \Omega$, $\mathbb{P}(\omega) = \frac{1}{|\Omega|}$

> For an *event* $E \subseteq \Omega$, $\mathbb{P}(E) = \frac{|E|}{|\Omega|}$

examples of uniform probability spaces:

Let your sample space be all possible outcomes of a sequence of 100 coin tosses. Assign the uniform measure to this sample space. What is the probability of the event "there are exactly 50 heads?

What is the sample space?

What is the probability measure?

What is the event we are interested in?

What is the probability of that event?

⭐***If we want to use uniform probability space, pick a sample space where every outcome is equally likely!***

Suppose you roll two dice. Each die is fair and they don't affect each other. What is the probability of both dice being even?

What is the sample space?

What is the probability measure?

What is the event we are interested in?

What is the probability of that event?

⭐***There is often more than one sample space possible! But one is probably easier than the others. Finding a sample space that will make the uniform measure correct will usually make finding the probabilities easier to calculate.***