even *more* counting

LECTURE 3

STARS AND BARS: C(n+k-1, n) ways to distribute n <u>indistinguishable</u> objects into k groups

COMBINATORIAL PROOF: Proof by showing that both sides of an equation count the same thing .

PIGEONHOLE PRINCIPLE: If you place n pigeons into k holes, at least $\lceil n/k \rceil$ pigeons in same hole.

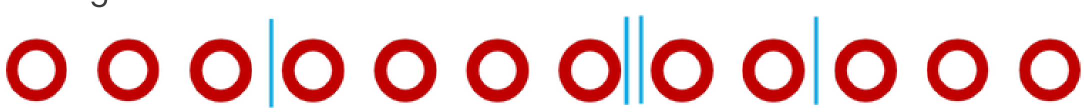
TODOS FOR WEEK 2

- ☐ Concept check 3 (due Wed 12pm)
- ☐ Concept check 4 (due Friday 12pm)
- ☐ Concept check 5 (due next Mon 12pm)
- ☐ HW1 due Wed 11:59pm ☐ start HW2
- ☐ Attend/participate in section (Thurs)

STARS AND BARS

The number of ways to distribute n <u>indistinguishable</u> objects into k groups is C(n+k-1, n)

e.g., How many ways to pick 12 donuts from 5 distinct flavors where donuts of the same type are indistinguishable and order of the donuts in the box does not matter?



used when picking objects from a groups where objects of the same type are ***indistinguishable***

------------------------PROOFS USING COUNTING TECHNIQUES------------------------

COMBINATORIAL PROOFS

Proofs by showing that <u>both sides of an equation count the same thing</u> (instead of algebraically)

e.g., prove symmetry of combinations $\binom{n}{k} = \binom{n}{n-k}$

e.g., prove Pascal's Rule $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

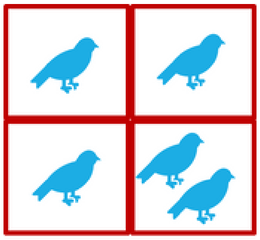
General Steps:

1. Describe a scenario
2. Explain how the LHS counts the outcomes in that scenario
3. Explain how the RHS counts the outcomes in that same scenario
4. State that "because the LHS and RHS both count the number of outcomes in the same scenario, they must be equal"

PIGEONHOLE PRINCIPLE

If you place n pigeons into k holes, and $n>k$, at least 2 pigeons are in the same hole.

If you place n pigeons into k holes, at least $\lceil n/k \rceil$ pigeons are in the same hole.



e.g., you have 10 classes, and have 3 quarters to take them in, then…

General Steps:

1. What are the pigeons? 2. What are the pigeonholes? 3. How to map from pigeons to holes?
4. There are n pigeons and k pigeonholes, so by pigeonhole principle, there is at least 1 pigeonhole with at least $\lceil n/k \rceil$ pigeons. This is what we're trying to prove because…

COUNTING REVIEW + PRACTICE!

Sum rule (split into disjoint sets)
Product rule (use a sequential process)
Complementary Counting (counting the ways for something to not occur)
Combinations (order doesn't matter)
Permutations (order does matter)

Principle of Inclusion-Exclusion (counting the size of a *union* of sets)
"**Stars and Bars**" (assign *indistinguishable* stars to distinguishable bins)
Niche Rules - Binomial Theorem (expanding (x+y)^n) and Pigeonhole Principle

Standard Deck of Cards (52 cards: 13 · 4=52)
Each card has one of **4 suits (♦, ♥, ♧, ♠)** and one of **13 values/ranks**

How many five-card "flushes" are there? A flush is a hand of cards all of the same suit.

How many 5-card hands have at least 3 aces?

Approach 1:
Step 1. Choose 3 aces (C(4,3)) **Step 2**. Pick 2 of the 49 remaining cards (C(49,2))
This approach _______________

How do we fix this solution?

HOW TO CHECK IF WE COUNTED CORRECTLY? (SLEUTH'S CRITERION)

For each outcome that we want to count, there should be exactly one set of choices in the sequential process that will lead to that outcome.
> If there are no sequence of choices that will lead to the outcome, we have *undercounted*.
> If there is more than one sequence of choices that will lead to the outcome, we *overcounted*.