more counting

LECTURE 2

PERMUTATIONS: *P(n,k) ways to make an <u>ordered</u> sequence of k from a group of n*

COMBINATIONS: *C(n,k) ways to make an <u>unordered subsets</u> of k from a group of n*

INCLUSION-EXCLUSION:
|A ∪ B ∪ C ∪...| = singles - doubles + triples - ....

Applications/Extensions:

BINOMIAL THEOREM: $(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$

Common Counting Strategy:
1. count simplifed problem (e.g., order matters)
2. divide/subtract out overcounting

K-PERMUTATIONS (ORDER MATTERS)

The number of k-element sequences of distinct symbols from a universe of n symbols is: P(n,k)

e.g., How many ways to arrange 3 people from a group of 20 people in a line for a picture?

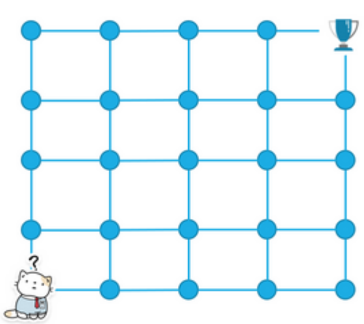
K-COMBINATIONS (ORDER DOESN'T MATTER)

The number of k-element subsets from a set of n symbols is: C(n,k)=P(n,k)/k!

e.g., How many ways to arrange 3 people from a group of 20 people to stand in a line for a picture?

Where does this formula come from?

PATH COUNTING



How many different paths are there? Each step is either right or up.

BINOMIAL THEOREM

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$$

Used when expanding a polynomial; finding coefficient of a term
For each monomial, $\binom{n}{i}$ ways to choose which n terms the x is from and $\binom{n-i}{n-i}$=1 ways to choose the which terms the y is from

REARRANGING WITH SOME DUPLICATES

How many anagrams are there of SEATTLE?

1st (incorrect) approach:
7! because rearranging 7 letters?

but this counts:
S**E**ATTLE and SEATTL**E** and different

2nd (correct) approach:
1. Pretend all the letters are distinct (e.g., E1 different from E2):

2. Divide out overcounting.

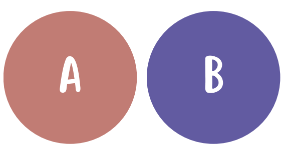
Useful counting technique:
With a more complex problem that can't be directly solved with one of the counting rules, we might:
1. Count options of a simplified version of problem
2. Divide out/subtract any overcounted outcomes

3rd (correct) approach:
1. Pick positions for the 2 E's:
2. Pick positions for the 2 T's:
3. Pick letters for remaining positions:

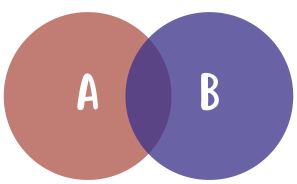
S T A E T L E

ANOTHER COUNTING RULE

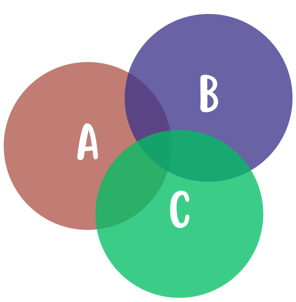
PRINCIPLE OF INCLUSION-EXCLUSION



SUM RULE:
Number of ways for A and B if they don't overlap is |A| + |B|
$|A \cup B| = |A| + |B|$ if $|A \cap B| = 0$



What if they overlap?
$|A \cup B| = |A| + |B| - |A \cap B|$
remove overcounting



What if there are 3 sets?
$|A \cup B \cup C| =$
$|A|+|B|+|C|-|A\cap B|-|B\cap C|-|A\cap C|+|A\cap B\cap C|$

in general....
$|A \cup B \cup C|$ = singles - doubles + triples - ....

notes...

How many length 5 strings over the alphabet $\{a,b,c,...,z\}$ contain:
Exactly 2 'a's OR Exactly 1 'b' OR No 'x's ---- *the "OR" hints at a union!*

Step 1: Define relevant sets: A ~ set of length 5 strings with exactly 2 a's, B ~ set of length 5 strings with exactly 1 b, C ~ set of length 5 strings with no x's

Step 2: Write what we're looking for: $|A \cup B \cup C|$ --> we can use inclusion-exclusion!

Step 3: List out all terms we need to compute, solve, and plug back into expression

$|A|$ = _____ $|A\cap B|$ = _____ $|A\cap B\cap C|$ = _____

$|B|$ = _____ $|B\cap C|$ = _____

$|C|$ = _____ $|A\cap C|$ = _____