

Homework 1

Due: Wednesday, June 26, by 11:59pm

Instructions

See [the instructions and FAQ for homeworks on the course website](#) for important notes on the submission format!

You must submit your solution via Gradescope. In particular:

- Submit a *single* PDF file containing the solution to all **written** tasks in the homework on Gradescope to **“HW 1 [Written]”**. Each numbered task should be solved on its own page (or pages). Follow the prompt on Gradescope to link tasks to your pages. Submit your coding assignment on Gradescope to **“HW 1 [Coding]”**.
- Do not write your name on the individual pages – Gradescope will handle that.

Task 0 – Collaborators

[0 pts]

List the full names of anyone you collaborated with on this homework. If you did not collaborate with anyone, write “None” in this section.

Task 1 – Video Walkthrough (nothing to submit here)

[0 pts]

Because of the odd schedule for this first homework, we’ve prepared a walkthrough video ([Video Part 1](#), [Video Part 2](#)) going through a homework style counting problem. This video also shows how much work you are expected to do on a similar problem. The problem we walk through in the video is below:

You’re at an ice cream shop where we can choose between having no cone, a sugar cone, or a waffle cone. You then pick 3 distinct ice cream scoops from a total of 10 available flavors. You like all flavor combinations except ones that have both mint and strawberry. Your last constraint is that if you choose chocolate as one of the scoops, you must have a waffle cone. The order in which the ice cream flavors are placed on the cone does not matter. How many possible ice cream orders do you have?

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Solution

The amount of work shown here would get full credit. Make sure to specify which rules you're applying, and how they are being applied.

We have two disjoint cases here:

Case 1 - Chocolate is NOT one of the flavors. We use the product rule here with the following sequential process:

1. Pick cone type - 3 options for this
2. Pick flavor combination - There are $\binom{9}{3}$ flavor combinations (choosing 3 distinct flavors from a set of 9 where order doesn't matter because there are 9 flavors excluding chocolate). Using complementary counting, we subtract the flavor combinations that have both mint and strawberry - there are 7 remaining flavors to pick from as the 3rd flavor. So, there are $\binom{9}{3} - 7$ options in this step.

Using the product rule, there are $3 \cdot (\binom{9}{3} - 7)$ options in this case.

Case 2 - Chocolate IS one of the flavors.

1. Pick cone type - 1 option (waffle cone)
2. Pick flavor combination - There are $\binom{9}{2}$ flavor combinations for the remaining 2 flavors. Using complementary counting, we subtract the flavor combinations that have both mint and strawberry - there are 1 option because the 3rd flavor must be chocolate. So, there are $\binom{9}{2} - 1$ options in this step.

Using the product rule, there are $1 \cdot (\binom{9}{2} - 1)$ options in this case.

Because these cases are disjoint, we use the sum rule, giving us $3 \cdot (\binom{9}{3} - 7) + 1 \cdot (\binom{9}{2} - 1)$ total options.

Task 2 – Syllabus

[5 pts]

Read the Syllabus, especially the collaboration policy, late policy, and the policy on exam conflicts. If you have questions, please ask them on Ed! Watch [the syllabus walkthrough video linked on the website](#). On your homework submission write "I have read the syllabus, and agree to follow the collaboration policies." for full credit.

Task 3 – Softball

[10 pts]

Fourteen people (6 children and 8 adults) on a softball team show up for a game.

- a) (3 points) How many ways are there to choose 4 players to take the field (order of players selected doesn't matter)?
- b) (3 points) How many ways are there to assign 4 players to the positions of catcher, pitcher, 1st baseman and shortstop by selecting players from the 14 people who show up?
- c) (4 points) How many ways are there to choose 4 players to take the field where at least one of these players must be an adult?

As described in the homework description, remember to explain your reasoning **for all problems** – do not just give numbers or unjustified calculations.

Task 4 – String Cheese

[24 pts]

We want to count the number of strings (an ordered sequence of letters) of length 4 from the English alphabet $\{A, B, \dots, Z\}$ subject to a number of different constraints. Note that we consider the English alphabet here to consist of 6 *vowels* ($\{A, E, I, O, U, Y\}$) and 20 *consonants*.

How many strings are there which ...

- a) ... are only made of vowels?
- b) ... are only made of consonants?
- c) ... have *exactly* one vowel?
- d) ... have *exactly* two vowels?
- e) ... have at most two vowels, which may only appear in the third and fourth position?
- f) ... have at least one vowel?

Task 5 – Food Festival

[10 pts]

You're organizing a potluck party with dishes from 4 different cuisines: Italian, Thai, Lebanese, and Nigerian. Each cuisine has 10 distinct dishes available. How many ways can you select a plate of 5 distinct dishes to taste, ensuring that you try at least one dish from each cuisine? The order of dishes doesn't matter.

Hint: Think about the number of dishes from each cuisine that will be selected.

Task 6 – Stuck in the Labyrinth

[20 pts]

This problem will make more sense after lecture on Friday. If you want to get started early, feel free to take a look at Friday's lecture slides.

In this problems, you will consider paths on the integer grid that start at $(0,0)$ in which every step increments one coordinate by 1 and leaves the other unchanged.

- a) (4 points) How many such paths are there from $(0,0)$ to $(85, 65)$?
- b) (4 points) How many such paths are there from $(0,0)$ to $(85, 65)$ that go through $(10,35)$?
How many such paths if they must go through $(15,40)$ instead?
- c) (6 points) How many such paths are there from $(0,0)$ to $(85, 65)$ that go through $(10,35)$, but do *not* go through $(15,40)$.
- d) (6 points) How many such paths from $(0,0)$ to $(85, 65)$ are there that go through neither of $(10,35)$ nor $(15,40)$ (i.e., that do not go through $(10,35)$ **and** do not go through $(15,40)$)?

Hint: Another way to rephrase this problem is "how many paths from $(0,0)$ to $(85,65)$ that do **not** go through either of $(10,35)$ **or** $(15,30)$ ".

Task 7 – How Many Pawsibilities?

[12 pts]

This problem will make more sense after lecture on Friday. If you want to get started early, feel free to take a look at Friday's lecture slides.

How many different ways are there to arrange the letters in the following words?

(a) dachshund (b) chihuahua

Task 8 – Don't Expand by Hand!

[15 pts]

The content in this question will only be covered on Friday in lecture. If you want to get started early, feel free to take a look at Friday's lecture slides.

The binomial theorem is very useful in many contexts! For example, in computer graphics, Bézier curves are used to draw complex shapes, and these are represented with polynomial equations. The binomial theorem is useful for efficiently expanding and evaluating these polynomials. In this problem, we'll work with some examples applying the binomial theorem.

- a) (7 points) What is the coefficient of x^4y^6 in the expansion of $(x - 3y^3)^6$? Do **not** use a calculator or try expanding this by hand! You must use the binomial theorem.
- b) (8 points) Use the binomial theorem to prove that

$$\sum_{i=0}^{500} \binom{500}{500-i} (-3)^i = 2^{500}$$

Hint: Start by trying to get the left side into a form so the binomial theorem can be applied.

Task 9 – A gentle introduction to Python

[15 pts]

1. **[Coding]** (10 points) Read the [HW Coding](#) lesson on Edstem and follow the directions to complete 5 coding exercises. Then submit the 5 required files from each exercise to **HW 1 [Coding]** on Gradescope. You may resubmit as many times on Gradescope before the deadline, and the score that appears on Gradescope for this part is final.
2. **[Written]** (5 points) Read the [Edstem lesson](#) on Python's numpy library, after completing the previous part. You do **not** need to complete any coding exercises or submit anything to Gradescope for this part. The exercise that is there is entirely OPTIONAL, and intended only for practice if you need it. Afterwards, write down what you felt was the most confusing numpy function and/or class to you and why. If nothing is confusing, explain which function and/or class is the most interesting to you. We will grade based on completion and effort rather than correctness, and it's recommended that your answer be no longer than 5 sentences.