SECTION EIGHT

Tail bounds + other stuff :D

MORE CONDITIONALS!

CONDITIONAL DISTRIBUTIONS

Sometimes we want to define a distribution for something like X|Y
- the random variable X given we know the value of the RV Y

	Discrete	Continuous
Conditional PMF/PDF	$p_{X\|Y}(x\|y) = \frac{p_{X,Y}(x,y)}{p_Y(y)}$	$f_{X\|Y}(x\|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$
Conditional Expectation	$\mathbb{E}\left[X\|Y=y\right] = \sum_x x p_{X\|Y}(x\|y)$	$\mathbb{E}\left[X\|Y=y\right] = \int_{-\infty}^{\infty} x f_{X\|Y}(x\|y)dx$

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A lot of this is similar to what you've seen already :)

CONDITIONAL DISTRIBUTIONS

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Linearity of expectation still applies to conditional distributions!

$$\mathbb{E}[X+Y|A] = \mathbb{E}[X|A] + \mathbb{E}[Y|A]$$

MORE

CONTINUOUS!

CONTINUOUS LOTP

This is the law of total probability for continuous random variables!

Instead of conditioning on a set of discrete events, and finding the probability of a discrete random variable, we might want to find the probability of __continuous__ random variables, partitioning on the values in the range of a different __continuous__ random variable.

$$\mathbb{P}(A) = \int_{x \in \Omega_X} \mathbb{P}(A | X = x) f_X(x) dx$$

LAW OF TOTAL EXPECTATION

Computed the expected value by partitioning on a set of events.

This is very similar in idea to law of total probability except that we're doing it for expectation.

$$\mathbb{E}[X] = \sum_{i=1}^{n} \mathbb{E}[X|A_i]\,\mathbb{P}(A_i)$$

$$\mathbb{E}[X] = \sum_{y} \mathbb{E}[X|Y=y]\,p_Y(y)$$

CONTINUOUS LOTE

$$\mathbb{E}[X] = \int_{y\in\Omega_Y} \mathbb{E}[X|Y=y]\,f_Y(y)dy$$

Same thing as above, but for continuous RVs!

BOUNDS

Markov + Chebyshev + Chernoff

If we find something like P(..) <= ..

—UPPER BOUND

If we find something like P(..) >= ..

—LOWER BOUND

TAIL BOUNDS

MARKOV

Gives an *upper bound* for P(X >= t):

$$\mathbb{P}\left(X \geq \alpha\right) \leq \frac{\mathbb{E}\left[X\right]}{\alpha}$$

<u>Requirements</u>
- X must be non-negative
- Must know E[X]

CHEBYSHEV

Gives an upper bound for:

$$\mathbb{P}\left(|Y-\mu| \geq \alpha\right) \leq \frac{\sigma^2}{\alpha^2}$$

<u>Requirements</u>
- Y can be *any* RV
- Must know E[X] *and Var(X)*

CHERNOFF

Gives *upper bound* for either tail:

$\mathbb{P}\left(X \geq (1+\delta)\,\mu\right) \leq e^{-\frac{\delta^2\mu}{3}}$ *–right tail*

$\mathbb{P}\left(X \leq (1-\delta)\,\mu\right) \leq e^{-\frac{\delta^2\mu}{2}}$ *– left tail*

<u>Requirements</u>
- X is sum of independent Bernoulli
- Must know E[X]

UNION

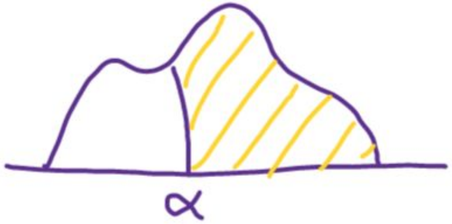
Gives *upper bound* for union of events:
“bound P of at least one....”
“bound P of even one urn having a ball”

$$P\left(\bigcup_{i=1}^{N} A_i\right) \leq \sum_{i=1}^{N} P(A_i)$$

TAIL BOUNDS

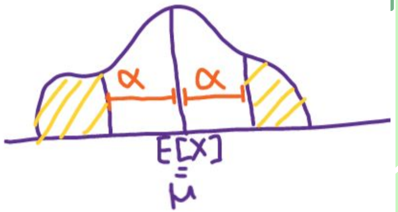
MARKOV

$\mathbb{P}(X \geq \alpha)$



CHEBYSHEV

$\mathbb{P}(|Y - \mu| \geq \alpha)$



CHERNOFF

$\mathbb{P}(X \geq (1+\delta)\mu) \leq e^{-\frac{\delta^2 \mu}{3}}$ *–right tail*

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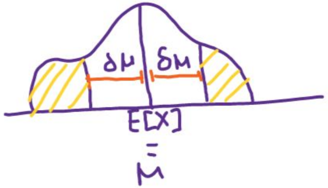

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