

Section 6

zoo of random variables :)

Random var.

discrete

(FINITE RANGE)



the range has discrete
values

two "types" of
random vars

discrete

(FINITE RANGE)



the range has discrete
values

two "types" of
random vars

continuous

(INFINITE RANGE)



sometimes the outcome is not
discrete (*for example time is not
discrete*)

discrete

(FINITE RANGE)



the range has discrete values

PMF (prob. mass function)

$$p_x(k) = P(X=k)$$

two "types" of random vars

continuous

(INFINITE RANGE)



sometimes the outcome is not discrete (*for example time is not discrete*)

discrete

(FINITE RANGE)



the range has discrete values

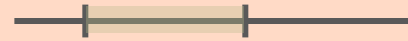
PMF (prob. mass function)

$$p_x(k) = P(X=k)$$

two "types" of random vars

continuous

(INFINITE RANGE)



sometimes the outcome is not discrete (*for example time is not discrete*)

PMF (prob. mass function)

$$p_x(k) = P(X=k) = 0$$

discrete

(FINITE RANGE)



the range has discrete values

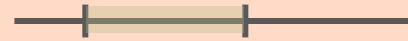
PMF (prob. mass function)

$$p_x(k) = P(X=k)$$

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PDF (prob. **density** function)

$$f_x(k) \neq P(X=k)$$

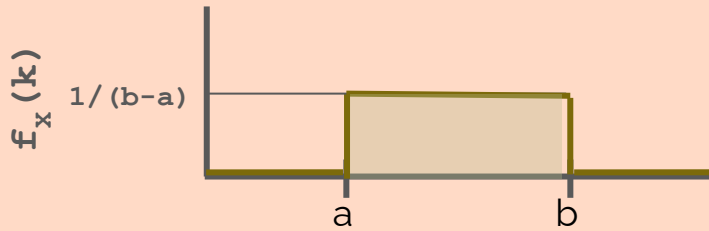
discrete vs. continuous

	Discrete	Continuous
PMF/PDF	$p_X(x) = P(X = x)$	$f_X(x) \neq P(X = x) = 0$
CDF	$F_X(x) = \sum_{t \leq x} p_X(t)$	$F_X(x) = \int_{-\infty}^x f_X(t) dt$
Normalization	$\sum_x p_X(x) = 1$	$\int_{-\infty}^{\infty} f_X(x) dx = 1$
Expectation	$\mathbb{E}[g(X)] = \sum_x g(x) p_X(x)$	$\mathbb{E}[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$

Zoo of continuous RVS

Uniform RV (continuous version)

$X \sim \text{Unif}(a, b)$ randomly takes on any real number between a and b



$$f_X(x) = \begin{cases} \frac{1}{b-a} & \text{if } x \in [a, b] \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbb{E}[X] = \frac{a+b}{2}$$

$$\text{Var}(X) = \frac{(b-a)^2}{12}$$

Exponential RV

$X \sim \text{Exp}(\lambda)$ tells how much time till a certain event happens
(λ is the rate of time)

think of this as the “continuous version”
of the geometric distribution!

don't confuse this with the Poisson
distribution just bc it's related with
time, they're very different!

(Poisson is *number* of events in a certain time frame)

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbb{E}[X] = \frac{1}{\lambda}$$

$$\text{Var}(X) = \frac{1}{\lambda^2}$$

$$F_X(x) = 1 - e^{-\lambda x}$$

$F_X(x) = P(X \leq x)$ this is the integral of $f_X(x)$