CSE 312

SECTION 5

ZOO OF RANDOM VARIABLES

– *Welcome back, everyone!* –

AGENDA

01 ANNOUNCEMENTS

02 LoE Reminder

03 ZOO OF RANDOM VARIABLES

02

LOE REMINDER

LOE

When working with linearity of expectation, remember to

first define the RVs and the summation relationships
don't worry how the individual RVs are distributed

then apply linearity of expectation and find each value

02

ZOO OF RV'S

zoo of discrete random variables!

Zoo of Discrete Random Variables

Random variables allow us to represent different random experiments/situations

We've seen how tedious computing pmfs, expectations, and variances can be.

There are some *common situations* that call for a random variable that is too complex to analyze, so we derive pmfs, expectations, and variance for this *"zoo" of RVs*.

UNIFORM

models situations where each outcome is equally likely

X ~ Uniform(a, b) if X is equally likely
to take on any value between a and b

$$p_X(k) = \frac{1}{b-a+1} \qquad \mathbb{E}[X] = \frac{a+b}{2} \qquad Var(X) = \frac{(b-a)(b-a+2)}{12}$$

A random variable X representing the outcome of rolling a fair 6 sided dice

X~Uniform(1, 6)

choosing a random value between 1 and 6 with each outcome equally likely

Bernoulli

models situations where the RV can take on 0 or 1 (whether success or not)

X ~ Bernoulli(p) if X is 1 with probability of p

$$p_X(k) = \begin{cases} p, & k = 1 \\ 1-p, & k = 0 \end{cases} \qquad \mathbb{E}[X] = p \qquad Var(X) = p(1-p)$$

X represents whether outcome of rolling a fair 6 sided dice is even (1) or not (0)

X~Bernoulli(3/6)

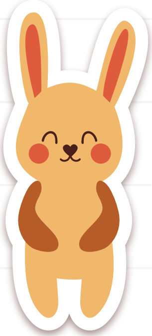
probability of 3/6 for "success"

Binomial

models situations when we count the # times an event occurs in n tries

X ~ Binomial(n, p) means X represents the number of times an event with probability p happens after n trials

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k} \qquad \mathbb{E}[X] = np \qquad Var(X) = np(1-p)$$



X represents the number of times the dice rolled to a 6 during 9 dice rolls

X~Binomial(⅙, 9)

probability of success (rolling a 6) on a single dice roll is ⅙, and 9 trials (rolls)

Geometric

models situations when we count the # trials until some event occurs

X ~ Geometric(p) means X represents the number of trials before success (an event with probability p happens)

$$p_X(k) = (1-p)^{k-1}p, \qquad \mathbb{E}[X] = \frac{1}{p} \qquad Var(X) = \frac{1-p}{p^2}$$

X represents the number of times we roll a 6 sided die, before it rolls a 6

X~Geometric(⅙)

on a single dice roll, there's a probability of ⅙ for success (that it rolls a 6)

NEGATIVE BINOMIAL

(RELATED TO GEOMETRIC)

MODELS SITUATIONS WHERE WE COUNT # TRIALS TO GET SOME NUMBER OF SUCCESSES

X ~ NegBin(r,p) means X represents the number of trials to get r successes (probability of success on a single trial is p)

$$p_X(k) = \binom{k-1}{r-1} p^r (1-p)^{k-r} \qquad \mathbb{E}[X] = \frac{r}{p} \qquad Var(X) = \frac{r(1-p)}{p^2}$$

X represents number of dice rolls before we get 4 rolls with a 6

X~NegBin(4, 1/6)

because we want to have 4 trials (rolls) with success, and a success (rolling 6) has probability 1/6

Poisson

Models situations with *time* - how many successes in a unit of time

X ~ Poisson(λ) means X represents the number of success in a unit of time, where λ is average rate of successes per unit of time

$$p_X(k) = e^{-\lambda}\frac{\lambda^k}{k!},$$

$$\mathbb{E}[X] = \lambda$$

$$Var(X) = \lambda$$

X represents number of people born during a particular minute

X~Poisson(λ)

where λ represents the average birth rate per minute

HYPERGEOMETRIC

Models situations with *choosing* - how many "successes" do you get when choosing without replacement

X ~ HypGeo(N,K,n) means X represents the number of successes out of n draws from N items with K successes

Number of ways you can choose n items with k successes

$$p_X(k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}} \qquad \mathbb{E}[X] = n\frac{K}{N} \qquad Var(X) = n \cdot \frac{K(N-K)(N-n)}{N^2(2N-1)}$$

Number of ways you can choose n items from N

X represents number of Kit-Kats we will get when drawing 30 candies from a bowl of 100 candies that contain 10 Kit-Kats

X~HypGeo(100, 10, 30)

because we draw 30 from 100 items with 10 successes (Kit-Kats)

LET'S TRY IT!

Let's identify these distributions in some real examples!

THANKS!