



CSE 312

Section 4



Random Variables





Random Variables



01

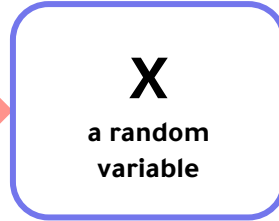
02

03

04



An outcome
from a random
experiment



Some number
(the **support of X** is the set of
possible values X can take on)

Probability Mass Function (PMF)

$P(X=k)$

probability that the random variable X will take on
the value k

what is the probability of an outcome that will result in X being k

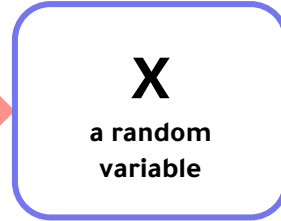
for discrete random variables (random variables with a finite, countably infinite range), this may sometimes be a piecewise function



Random Variables



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Cumulative Distribution Function

$$F_X(k) = P(X \leq k) \rightarrow \text{probability that the value } X \text{ takes on is less than or equal to } k$$

what is the probability of an outcome that will result in X being $\leq k$

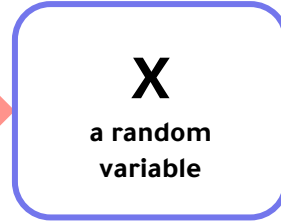
often can be derived from the PDF



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Expectation

$$E[X] = \sum(k \cdot P(X=k))$$

sum of values in the range of X,
weighted by the probability
on average, what value can we "expect" X to take?

think about it like a weighted average of all the possible values X could be (weighted by the $P(X=k)$)



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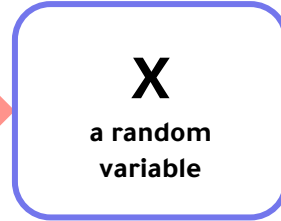




Random Variables



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Expectation

$$E[X] = \sum(k \cdot P(X=k))$$

sum of values in the range of X,
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on average, what value can we "expect" X to take?

just averaging all the possible values of X wouldn't work since each outcome isn't necessarily equally likely



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INDEPENDENT RV

What does independence mean for
random variables?





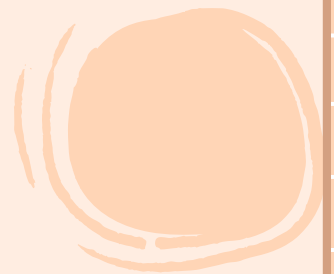
Random Variable Independence



Random variables X and Y are **independent** if –

$$\mathbf{P(X=x, Y=y) = P(X=x) \cdot P(Y=y)}$$

Knowing the value of X doesn't help "guess" what Y is





Random Variable Independence



Random variables X and Y are **independent** if –

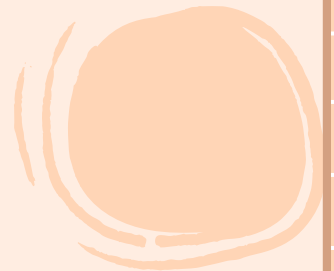
$$P(X=x, Y=y) = P(X=x) \cdot P(Y=y)$$

Knowing the value of X doesn't help "guess" what Y is

it's a useful property! if X and Y are independent random variables then –

$$E(X \cdot Y) = E[X] \cdot E[Y]$$

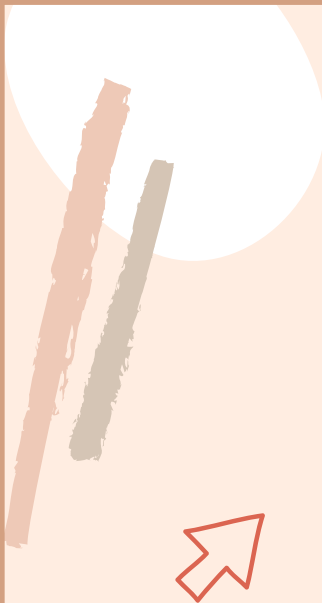
$$\text{Var}(X + Y) = \text{Var}[X] + \text{Var}[Y] \quad \textit{Linearity of variance holds}$$





Variance

Variance is another property of RVs (like expectation) that measures how much the values in the RV “vary”





03 - Variance



Random Variables

allow us to represent a quantitative property of a random experiment

VARIANCE - how “different” are values from the expectation “on average”

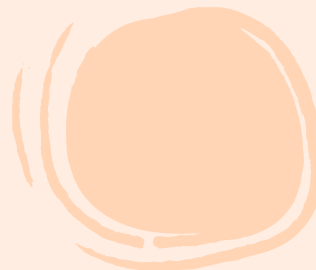
every random variable has some variance

$$\text{Var}(X) = E[(X - E(X))^2] = \sum_x (P(X=x) * (x - E(X))^2)$$

expected value of the squared distance between each RV outcome and the expected value of RV

add up all the squared distances weighted by their probabilities

$$\text{variance} = (\text{standard deviation})^2$$





03 - Variance



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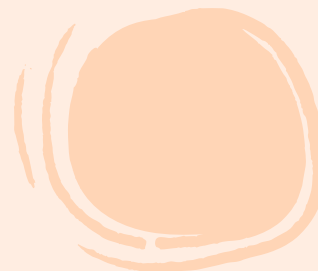
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$$\text{Var}(X) = E[(X - E(X))^2] = \sum_x (P(X=x) * (x - E(X))^2)$$

Properties

$$\text{Var}(a \cdot X + b) = a^2 \cdot \text{Var}(X)$$

$$\text{Var}(X) = E[X^2] - (E[X])^2$$





LoE

Linearity of Expectation is a powerful property of random variables!

Will be covered on Friday in lecture!



02 - Linearity of Expectation



Random Variables

allow us to represent a quantitative property of a random experiment

EXPECTATION - weighted average of possible outcomes

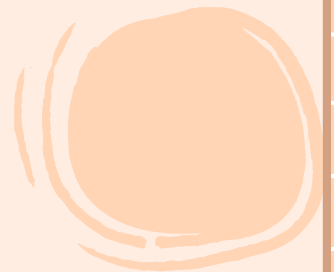
you could use “brute force” and use the formula for expectation ($E[X] = \sum (x * P(x))$)

sometimes, just applying the formula can be messy, so LoE comes in handy

LINEARITY OF EXPECTATION (LoE) *is one important property*

$$E(X+Y) = E(X) + E(Y)$$

the expected value of the sum of 2 random variables is
the sum of their expected values





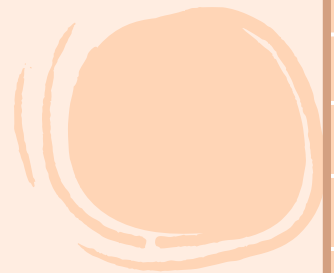
02 - Linearity of Expectation



$$\mathbf{E(X+Y) = E(X) + E(Y)}$$

the expected value of the sum of 2 random variables is
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*this gives us a helpful **tool to calculate expectations of complex RVs***





02 - Linearity of Expectation



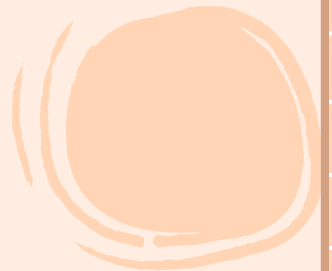
$$E(X+Y) = E(X) + E(Y)$$

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DECOMPOSE into a sum of random variables

$$X = X_1 + X_2 + \dots + X_n$$





02 - Linearity of Expectation



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APPLY linearity of expectation

$$E[X] = E[X_1] + E[X_2] + \dots + E[X_n]$$



02 - Linearity of Expectation



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DECOMPOSE into a sum of random variables

$$X = X_1 + X_2 + \dots + X_n$$

APPLY linearity of expectation

$$E[X] = E[X_1] + E[X_2] + \dots + E[X_n]$$

CONQUER and calculate each value

$$E[X_1] = \dots, E[X_2] = \dots, \dots$$

$$E(X+Y) = E(X) + E(Y)$$

the expected value of the sum of 2 random variables is the sum of their expected values

sometimes, these X_i variables we "decompose" X into are **indicator** random variables

this gives us a helpful tool to calculate expectations of complex RVs

DECOMPOSE into a sum of random variables

$$X = X_1 + X_2 + \dots + X_n$$

APPLY linearity of expectation

$$E[X] = E[X_1] + E[X_2] + \dots + E[X_n]$$

CONQUER and calculate each value

$$E[X_1] = \dots, E[X_2] = \dots, \dots$$



02 - Linearity of Expectation



Indicator Random Variables

we can define a *indicator random variable* X for an event A

$$X = \begin{cases} 1 & \text{if event } A \text{ happens} \\ 0 & \text{if event } A \text{ doesn't happen} \end{cases}$$

X tells us whether event A will happen \rightarrow so, $P(X = 1) = P(A)$

Note that $E[X] = 1 * P(X=1) + 0 * P(X=0) = P(X=1)$

*this is why indicator RVs
can be really useful when
applying linearity of
expectation!*



02 - Linearity of Expectation

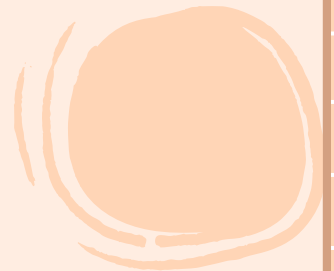


linearity of expectation is special!

$$E[X+Y] = E[X] + E[Y] \text{ but } E[X^2] \neq (E[X])^2$$

instead...

$$E[g(X)] = \sum (g(x) * P(X=x))$$





That's all for now!
Contact me at clarisw@uw.edu
if you have any questions!

Thanks for coming!

