cse 312
winter 2024

ENTER

week 3 :)

content review

ENTER

What we talked about this week

More about probability

Conditional probability

- Bayes' rule
- Law of total probability
- Chain rule

More about events

Independence

- Independence
- Conditional independence

Conditional Probability

Credit: Super Mario Wiki

Conditional Probability

01
02
03
04

$$P(A|B)$$

probability of the event A occurring
given that
the event B occurs

"what is the probability that event A happens given that event B happened?"

Conditional Probability

$$P(A|B)$$

probability of the event A occurring
given that
the event B occurs

"what is the probability that event A happens after learning that event B happened?"

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Conditional Probability

01

02

03

04

$$P(A|B)$$

probability of the event A occurring
given that
the event B occurs

"what is the probability that event A happens given that event B happened?"

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)\,P(A)}{P(B)}$$

–BAYES RULE–

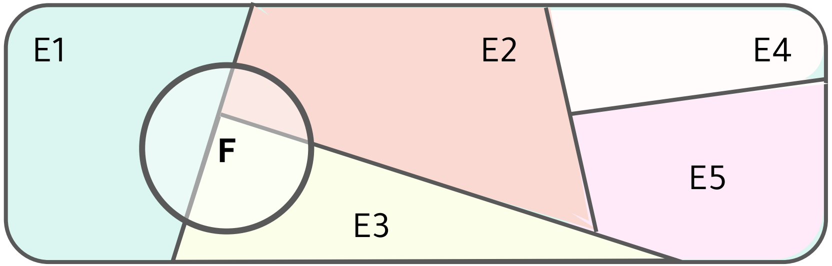
TIPS:
- start by writing all the probabilities you know
- write down what you want to find

Conditional Probability

We can use conditional probability to help calculate more complex probabilities!

Conditional Probability - LTP

we can *partition a sample space* into discrete events



Ω = E1 ∪ E2 ∪ E3 … E5

divided the set of all possible outcomes into "disjoint" event sets

The probability of any other event F that is inside of this sample space Ω is

by definition of cond. probability ->

$$P(F) = P(F \cap E1) + P(F \cap E2) \ldots + P(F \cap E5)$$

$$= P(F \mid E1)P(E1) + P(F \mid E2)P(E2) + \ldots + P(F \mid E5)P(E5)$$

–LAW OF TOTAL PROBABILITY–

Conditional Probability - Chain Rule

sometimes we have a **sequential process** and want to find the probability of that

e.g., finding the probability that event E1 happened, then event E2 happens, …. then event En happens

watch out for sometimes when the counting method may be easier

$$P(E1 \cap E2 \cap E3 \cap En) = P(E1) \cdot P(E2 \mid E1) \cdot P(E3 \mid E2 \cap E1) \cdot \ldots \cdot P(En \mid E1 \cap E2 \ldots \cap E(n-1))$$

—CHAIN RULE—

multiplying probability of each event happening conditioned on all the previous events

Connecting Chain rule, Bayes' Rule, & LTP

Chain Rule!

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|A^C)\,P(A^C)}$$

Law of Total Probability!

Independence

Independence

two events are independent if there is no correlation between the events and they don't depend on each other

two events A, B are statistically <u>independent</u> if

$$P(A \cap B) = P(A) \cdot P(B)$$

or

$$P(A \mid B) = P(A) \quad \textit{and} \quad P(B \mid A) = P(B)$$

"knowing that B happened doesn't affect the probability that A will happen and vice versa"
"knowing that B happened doesn't give any new information about A"

Just because 2 events may "sound like" they're independent, that doesn't mean that they are *statistically* independent

Conditional Independence

two events A, B are conditionally independent if

$$P((A \cap B) \mid C) = P(A \mid C) \cdot P(B \mid C)$$

Just because 2 events may "sound like" they're independent, that doesn't mean that they are *statistically* independent

Thank you!

OK